$\bar{x} = 122$
$\sigma = 20$
min = 30
med = 119
max = 150
81.3 % avg.

Test Total 150 pts

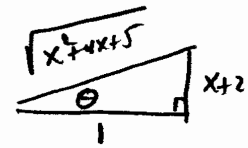
Name KEY

Final Exam **Calculus II** **Drs. Clemons & Norfolk**
May 8, 2006

Show all work. Partial credit will be given for correct reasoning.

(1) Evaluate the following integrals. State if it is an improper integral:

(a) $\int_0^1 \frac{1}{\sqrt{x^2+4x+5}}\,dx = \int_0^1 \frac{dx}{\sqrt{(x+2)^2+1}}$ 2 $\qquad x+2 = \tan\theta$ 2, $\quad dx = \sec^2\theta\, d\theta$



$= \int_{x=0}^{x=1} \frac{\sec^2\theta}{\sec\theta}\,d\theta$ 2 $= \ln|\sec\theta + \tan\theta|\Big|_{x=0}^{x=1}$ 2

$= \ln\left|\sqrt{x^2+4x+5} + x+2\right|\Big|_0^1 = \ln|\sqrt{10}+3| - \ln|\sqrt{5}+2| = .3748$

3 $\qquad$ 1

12 pts

(b) $\int_1^3 \frac{x+3}{x^2-3x}\,dx = \int_1^3 \frac{x+3}{x(x-3)}\,dx$ 1 $= \int_1^3 \frac{-1}{x} + \frac{2}{x-3}\,dx$ 2 2

$= \lim_{b\to 3^+} \left[-\ln x + 2\ln(x-2)\right]_1^b$ 2 $= \lim_{b\to 3^+}$ 3 $[-\ln b + 2\ln(b-3)] - [-\ln 1 + 2\ln|-1|]$

$= -\ln 3 - \infty = -\infty.$

2

12 pts

(c) $\int 3x2^x\,dx = \int 3x\, e^{\ln 2\cdot x}\,dx$ 2

$u = 3x \qquad dv = e^{\ln 2\cdot x}\,dx$ 4
$du = 3dx \qquad v = \frac{1}{\ln 2} e^{\ln 2 x}$

$\int e^{\ln(3x\cdot 2^x)}\,dx$
$= \int e^{\ln 3 + \ln x + \ln 2^x}$
$= \int 3\cdot x\; e^{x\ln 2}\,dx$

$= \frac{3}{\ln 2} x e^{\ln 2 x} - \int \frac{3}{\ln 2} e^{\ln 2 x}\,dx$ 2

$= \frac{3}{\ln 2} x e^{\ln 2 x} - \frac{3}{(\ln 2)^2} e^{\ln 2 x} + C$ 2

$= \left(\frac{3}{\ln 2} x - \frac{3}{(\ln 2)^2}\right) 2^x + C$ 2

12 pts

Pg 1 Tot: 36

Name ______________________________

(d) $\displaystyle\int \frac{\sec^2 t}{1-\tan t}\,dt$

$u = 1-\tan t$ (4)
$du = -\sec^2 t\,dt$

$= \int -\frac{1}{u}\,du$ (2)

$= -\ln|u| + C$ (3) $= -\ln|1-\tan t| + C$ (3)

$= \ln\left|\frac{1}{1-\tan t}\right| + C$

12 pts

(e) $\displaystyle\int \sin^3 x \cos^8 x\,dx$

$= \int \sin^2 x \cos^8 x \sin x\,dx$ (2)

$= \int (1-\cos^2 x)\cos^8 x \sin x\,dx$ (2)

$u = \cos x \quad du = -\sin x\,dx$ (2)

$= \int -(1-u^2)u^8\,du = \int u^{10} - u^8\,du$ (2)

$= \frac{1}{11}u^{11} - \frac{1}{9}u^9 + C$ (2)

$= \frac{1}{11}\cos^{11} x - \frac{1}{9}\cos^9 x + C$ (2)

12 pts

(f) $\displaystyle\int \sqrt{2-x^2}\,dx$

$x = \sqrt{2}\sin\theta, \quad dx = \sqrt{2}\cos\theta\,d\theta$ (2)

$= \int \sqrt{2-2\sin^2\theta}\cdot\sqrt{2}\cos\theta\,d\theta$ (2)

$= \int 2\cos^2\theta\,d\theta$ (2)

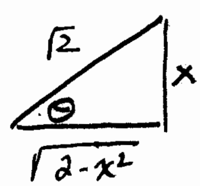
$= \int 1+\cos 2\theta\,d\theta$ (2)

$= \theta + \frac{1}{2}\sin 2\theta\,d\theta$ (2)

$= \theta + \sin\theta\cos\theta + C$

$= \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{x}{\sqrt{2}}\cdot\frac{\sqrt{2-x^2}}{\sqrt{2}} + C$ (2)

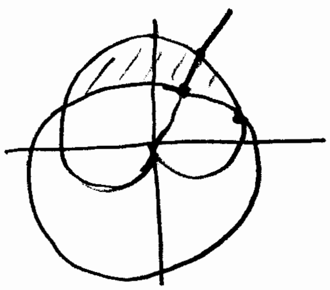
(2) (2)



12 pts

Pg 2 Tot: 36

(2) Sketch the region inside $r = 2 + 2\sin\theta$ and outside $r = 3$, then Set-up the integral representing the area. **Do Not Solve Your Integral.**



$r = r$

$3 = 2 + 2\sin\theta$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \int_{\frac{\pi}{6}}^{5\pi/6} \frac{1}{2}\left[(2+2\sin\theta)^2 - 3^2\right] d\theta$$

(2) (2) (2) (2) (2)

10 pts

(3) Convert the polar equation, $r = \dfrac{10}{3 - 2\cos\theta}$, to a cartesian equation and *identify the conic section, including any centers, major/minor axes, intercepts and asymptotes.*

$3r - 2r\cos\theta = 10$

$3\sqrt{x^2+y^2} - 2x = 10$

$3\sqrt{x^2+y^2} = (10+2x)^2$

$9(x^2+y^2) = 100 + 40x + 4x^2$

$5x^2 - 40x + 9y^2 = 100$ (4)

$5(x^2 - 8x \quad) + 9y^2 = 100$

$5(x^2 - 8x + 16 - 16) + 9y^2 = 100$

$5(x-4)^2 - 80 + 9y^2 = 100$

$5(x-4)^2 + 9y^2 = 180$

$\frac{(x-4)^2}{36} + \frac{y^2}{20} = 1$

$y = \pm\sqrt{\frac{100}{9}} = \frac{10}{3}$

4 ± 6

10 pts

(2) Ellipse! Center $(4,0)$

(2) minor axis : $\sqrt{20} = 2\sqrt{5}$

major axis : 6

(1)

(2) intercepts $(4 \pm 6, 0) = (-2,0)$ & $(10,0)$

$(0, \pm\frac{10}{3})$

( )

Pg 3 Tot: 20

Name ______________________________

(4) Given the parametric curve $x(t) = t^2$, $y(t) = t - \frac{1}{3}t^3$, for $0 \le t \le 1$,

(a) Calculate $\frac{d^2y(t)}{dx^2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \overset{2}{=} \frac{1-t^2}{2t} \overset{2}{=} \frac{1}{2t} - \frac{1}{2}t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \overset{3}{=} \frac{\frac{d}{dt}\left(\frac{1}{2t} - \frac{1}{2}t\right)}{2t} \overset{3}{=} \frac{\frac{-1}{2t^2} - \frac{1}{2}}{2t} = -\frac{1}{4t^3} - \frac{1}{4t}$$

10 pts

Set-up , but do not evaluate the integrals necessay to calculate

(b) the arclength for $0 \le t \le 1$.

$$L_0^1 = \int_0^1 \sqrt{x'^2 + y'^2}\, dt = \int_0^1 \sqrt{(2t)^2 + (1-t^2)^2}\, dt$$

2 2 2 2

8 pts

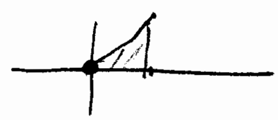
(c) the area of the surface generated by revolving the parametric curve about the x-axis.

$$SA = \int_0^1 2\pi y \sqrt{x'^2 + y'^2}\, dt = \int_0^1 2\pi \left(t - \tfrac{1}{3}t^3\right) \sqrt{(2t)^2 + (1-t^2)^2}\, dt$$

2 2 2 2

8 pts

(d) the area under the parameterized curve.



$$A = \int_0^1 y \frac{dx}{dt} \cdot dt = \int_0^1 \left(t - \tfrac{1}{3}t^3\right)(2t)\, dt$$

2 3 3

8 pts

Pg 4 Tot: 34

(4) Find the interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{\ln n}{n}(x-1)^n$. Determine the convergence or divergence at the endpoints.

5

Ratio TEST: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{\ln n+1}{n+1}(x-1)^{n+1}}{\frac{\ln n}{n}(x-1)^n}\right| = \lim_{n\to\infty}\left|\frac{\ln n+1}{\ln n}\right|\cdot\left|\frac{n}{n+1}\right||x-1|$

$= |x-1| < 1 \Rightarrow 0 \overset{?}{\leq} x \overset{?}{\leq} 2$ 3

Center: $x=1$

Radius: $R=1$

interval $[0,2)$

3 @ $x=0$: $\sum_{n=2}^{\infty}\frac{\ln n}{n}(-1)^n$ $\lim_{n\to\infty}\frac{\ln n}{n}=0$, $\left[\frac{\ln n}{n}\right]'<0$ Converges by Alt. Series TEST

3 @ $x=2$: $\sum_{n=2}^{\infty}\frac{\ln n}{n}$: $\frac{\ln n}{n} > \frac{1}{n}$ for $n>e$, so diverges by BCT.

12 pts

(5) Write a power series expansion, $\sum_{n=0}^{\infty} A_n x^n$ and state the radius of convergence for

(a) $\ln(1+x)$

$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n$

$\ln(1+x) = \sum_{n=0}^{\infty}(-1)^n \int x^n dx = \boxed{\sum_{n=0}^{\infty}(-1)^n \cdot \frac{1}{n+1}x^{n+1}}$, $|x|<1$.

4 pts

(b) $\frac{x}{3+2x}$ $= x \sum_{n=0}^{\infty}(-1)^n\frac{2^n}{3^{n+1}}x^n = \sum_{n=0}^{\infty}(-1)^n\frac{2^n}{3^{n+1}}x^{n+1}$, $|x|<\frac{3}{2}$

$\frac{1}{3+2x} = \frac{1}{3}\cdot\frac{1}{1+\frac{2}{3}x} = \frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{3}\right)^n x^n = \boxed{\sum_{n=0}^{\infty}(-1)^n\frac{2^n}{3^{n+1}}x^n}$ $\left|\frac{2}{3}x\right|<1$

$|x|<3/2$

4 pts

(c) $\ln(1+x) + \frac{x}{3+2x}$

$= \sum_{n=0}^{\infty}(-1)^n\frac{1}{n+1}x^{n+1} + \sum_{n=0}^{\infty}(-1)^n\frac{2^n}{3^{n+1}}x^{n+1}$

$\boxed{= \sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{n+1}+\frac{2^n}{3^{n+1}}\right]x^{n+1}}$ $|x|<1$

4 pts

Pg 5 Tot: 24