$\bar{X} = 82.4$
$\sigma = 13.6$
min=50
med = 81.5
max=100

Test 3 Calculus II 3450:222 April 14, 2006
Show all of your work.

1. For each of the following series, determine whether they *converge absolutely*, *converge conditionally*, or *diverge*. You must state which test(s) you are using, and show that you have checked all appropriate conditions.

(a) $\sum_{n=1}^{\infty} \frac{5-2n+6n^2}{\sqrt{5n^8+n^6+7}}$ $\approx \frac{n^2}{n^4} = \frac{1}{n^2}$ (2)

(3)

L.C.T. $\lim_{n\to\infty} \frac{\frac{5-2n+6n^2}{\sqrt{5n^8+n^6+n^7}}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{5n^2-2n^3+6n^4}{\sqrt{5n^8+n^6+n^7}} = \frac{6}{\sqrt{5}} < \infty.$

(1)

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series, by the L.C.T. $\sum_{n=1}^{\infty} \frac{5-2n+6n^2}{\sqrt{5n^8+n^6+7}}$ converges absolutely.

(1)

7 points

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln n}$

(3)

Assoc. pos. series $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$. NOTE $\frac{1}{n^2 \ln n} < \frac{1}{n^2}$, for $n > e$

(2)

Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ is a convergent p-series, $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln n}$ converges absolutely by B.C.T.

(2)

7 points

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{(2n)! n!}$

(2)

Ratio Test:

7 points

$\lim_{n\to\infty} \frac{\frac{(3(n+1))!}{(2(n+1))!\,(n+1)!}}{\frac{(3n)!}{(2n)!\,n!}} = \lim_{n\to\infty} \frac{(3n+3)!}{(3n)!} \cdot \frac{(2n)!}{(2n+2)!} \cdot \frac{n!}{(n+1)!}$

Page 1 (21)

(3)

$= \lim_{n\to\infty} (3n+3)(3n+2)(3n+1) \cdot \frac{1}{(2n+2)(2n+1)} \cdot \frac{1}{(n+1)}$

$= \frac{27}{4} > 1$, So by the Ratio TEST, the series diverges

(2)

(d) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\lim_{n\to\infty} \frac{1}{n\ln n} \to 0$, $\left[\frac{1}{n\ln n}\right]' < 0$ 2

I.T. $\int_2^\infty \frac{1}{n\ln n} = \int_{\ln 2}^{\infty} \frac{1}{u} du = \ln u \Big|_{\ln 2}^{\infty} = \infty$.
1 3

So by I.T., $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges.
1

7 points

(e) $\sum_{n=1}^{\infty} \left(\frac{1-3n}{5n+1}\right)^n$

Root Test: 4 2

$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{1-3n}{5n+1}\right)^n\right|} = \lim_{n\to\infty} \left|\frac{1-3n}{5n+1}\right| = \frac{3}{5} < 1$

So converges absolutely
1

7 points

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

3

assoc. pos series $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$. This is a divergent p-series w/ $p=\frac{1}{2}$.
2

However, $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ & $\left[\frac{1}{\sqrt{n}}\right]' < 0$, so by the alt. series Test, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges conditionally 2

7 points

(g) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n + (0.5)^n}$

$\lim_{n\to\infty} \frac{n}{2n+(0.5)^n} = \frac{1}{2} \neq 0$ 4

3 Series diverges by necessary condition of convergence, $\lim_{n\to\infty} a_n = 0$.

7 points

2. Use a geometric series to write 2.74 74 74 $\cdots$ as a rational number. 1

2 $2+\frac{74}{100}+\frac{74}{100^2}+\cdots = 2+\sum_{n=1}^{\infty}\frac{74}{100^n}$ 2 $= 2+74\sum_{n=0}^{\infty}\frac{1}{100}\cdot\frac{1}{100^n}$

$= 2+74\underbrace{\left(\frac{\frac{1}{100}}{1-\frac{1}{100}}\right)}_{3} = 2+\frac{74}{99} = \frac{272}{99}$

1 1

10 points

3. Find the *centre*, *radius of convergence* and *interval of convergence* for the power series $\sum_{n=0}^{\infty}\frac{(3-2x)^n}{4^n+1}$.

R.T.

$\lim_{n\to\infty}\left|\frac{\frac{(3-2x)^{n+1}}{4^{n+1}+1}}{\frac{(3-2x)^n}{4^n+1}}\right| = \lim_{n\to\infty}\frac{4^n+1}{4^{n+1}+1}\cdot|3-2x| = \frac{1}{4}(3-2x) < 1$ ← Impose

1 2 2

$|3-2x|<4$ or $\left|\frac{3}{2}-x\right|<2$. Radius $=2$, Centre $=\frac{3}{2}$

Interval: $-\frac{4}{3}<1-\frac{2}{3}x<\frac{4}{3} \Rightarrow -\frac{7}{3}<-\frac{2}{3}x<\frac{1}{3} \Rightarrow -\frac{1}{2}<x<\frac{7}{2}$

? ?

2 @ $-\frac{1}{2}$ $\sum_{n=0}^{\infty}\frac{4^n}{4^n+1}$ diverges, $\lim_{n\to\infty}\frac{4^n}{4^n+1}=1$

2 @ $\frac{7}{2}$ $\sum_{n=0}^{\infty}(-1)^n\frac{4^n}{4^n+1}$ diverges

Interval of convergence $\left(-\frac{1}{2}, \frac{7}{2}\right)$ 1

11 points

4. (a) Write the Maclaurin series for $\cos x$.

$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

3 2

5 points

(b) Use your series above to write a simplified power series for $\cos\sqrt{x}$.

$\cos\sqrt{x} = 1-\frac{x}{2}+\frac{x^2}{4!}-\frac{x^3}{6!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{(2n)!}$, $x\geq 0$

3 2

5 points

Page 3 (31)

(c) Use your power series to write $\int_0^1 \cos\sqrt{x}\,dx$ as a series.

$$\int_0^1 \cos\sqrt{x}\,dx = \sum_{n=0}^{\infty} \int_0^1 \frac{(-1)^n x^n}{(2n)!}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)(2n)!}\Bigg|_0^1 \quad 3$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)(2n)!} = \frac{1}{1} - \frac{1}{2\cdot 2} + \frac{1}{3\cdot 4!} - \frac{1}{4\cdot 6!}$$

2

5 points

5. Suppose that $f(x) = \sum_{n=0}^{\infty} \frac{2-\sin^2 n!}{4^n}(x-2)^n$.

Find $f(2)$, $f'(2)$ and $f^{(13)}(2)$.

1 $f(2) = a_0 = \frac{2-\sin^2 0!}{4^0} = 2$

1 $f'(2) = 1!\,a_1 = \frac{2-\sin^2 1}{4}$

3 $f^{(13)}(2) = 13!\,a_n = 13!\left(\frac{2-\sin^2 13!}{4^{13}}\right)$

5 points

6. Find the first 3 non-zero terms of the Maclaurin expansion for $g(x) = \ln(1+x) + e^x$.

1 $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$

1 $\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$

1 + $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ 1 $= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$

$\ln(1+x) + e^x = 1 + 2x + \left(\frac{1}{3}+\frac{1}{6}\right)x^3 + \cdots$

10 points

$= 1 + 2x + \frac{1}{2}x^3 + \cdots$

2 2 2