Test Total

Name KEY

Exam 2 Calculus II Dr. Clemons f96

I. Evaluate the following improper integrals. State whether the integral converges or diverges.

(1) $\int_2^{\infty} \frac{1}{x \ln x}\, dx = \lim_{b\to\infty} \int_2^b \frac{1}{x \ln x}\, dx$ let $u = \ln x$, $du = \frac{1}{x}dx$

$= \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{1}{u}\, du = \lim_{b\to\infty} \ln|u| \Big|_{\ln 2}^{b}$

$= \lim_{b\to\infty} \ln|b| - \ln(\ln 2) = \infty$, diverges.

13 pts

(2) $\int_0^{\pi/2} (\tan x - \sec x)^2\, dx$

$= \lim_{b\to\frac{\pi}{2}} \int_0^b (\tan x - \sec x)^2 dx = \lim_{b\to\frac{\pi}{2}} \int_0^b \tan^2 x - 2\sec x \tan x + \sec^2 x\, dx$

$= \lim_{b\to\frac{\pi}{2}} \int_0^b (\sec^2 x - 1) - 2\sec x \tan x + \sec^2 x\, dx$

$= \lim_{b\to\frac{\pi}{2}} \int_0^b -1 - 2\sec x \tan x + 2\sec^2 x\, dx$

$= \lim_{b\to\frac{\pi}{2}} -x - 2\sec x + 2\tan x \Big|_0^b$

$= \lim_{b\to\frac{\pi}{2}} (-b - 2\sec b + 2\tan b) - (-0 - 2 + 2(0))$

$= -\frac{\pi}{2} - 2(\infty) + 2\cdot\infty + 2$, $\underbrace{}_{\text{indet form}}$

$= -\frac{\pi}{2} + \lim_{b\to\frac{\pi}{2}} +2\left(\frac{\sin b - 1}{\cos b}\right) + 2$

L'H

$= -\frac{\pi}{2} + \lim_{b\to\frac{\pi}{2}} +2\left(\frac{\cos b}{\sin b}\right) + 2$

$= -\frac{\pi}{2} + 2$

13 pts

Pg 1 Tot: 26

II. Evaluate the following integrals. Partial Credit will be given only for application of appropriate methods.

(1) $\int \sqrt{\cos x}\sin^3 x\,dx$ $= \int \sqrt{\cos x}\,\sin^2 x \cdot \sin x\,dx = \int \sqrt{\cos x}\,(1-\cos^2 x)\cdot \sin x\,dx$

$u = \cos x \quad du = -\sin x\,dx$

$= \int \sqrt{u}\,(u^2-1)\,du = \int u^{5/2} - u^{1/2}\,du$

$= \frac{2}{7}u^{7/2} - \frac{2}{3}u^{3/2} + C$

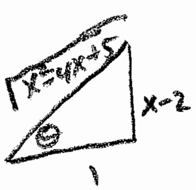
$= \frac{2}{7}(\cos x)^{7/2} - \frac{2}{3}(\cos x)^{3/2} + C$

11 pts

(2) $\int \frac{1}{\sqrt{x^2-4x+5}}\,dx$ $= \int \frac{1}{\sqrt{(x-2)^2+1}}\,dx$

$x-2 = \tan\theta \qquad dx = \sec^2\theta\,d\theta$

$= \int \frac{1}{\sqrt{\tan^2\theta+1}}\cdot \sec^2\theta\,d\theta$



$= \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\sqrt{x^2-4x+5} + x - 2\right| + C$

11 pts

Pg 2 Tot: 22

Name ______________________________

(3) $\int \frac{x^2+x+6}{x^3-3x^2}\,dx$

$$= \int \frac{x^2+x+6}{x^2(x-3)}\,dx \qquad \frac{x^2+x+6}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$= \frac{Ax(x-3)+B(x-3)+Cx^2}{x^2(x-3)}$$

$$= \int -\frac{1}{x} - \frac{2}{x^2} + \frac{2}{x-3}\,dx$$

$$= \frac{(A+C)x^2+(-3A+B)x-3B}{x^2(x-3)}$$

$$= -\ln|x| + \frac{2}{x} + 2\ln|x-3| + C$$

$A + C = 1 \qquad C = 2$

$-3A + B = 1 \qquad A = -1$

$-3B = 6 \qquad B = -2$

11 pts

(4) $\int e^{-x}\sin(2x)\,dx$

$u = e^{-x} \quad dv = \sin 2x\,dx$

$du = -e^{-x}dx \quad v = -\frac{1}{2}\cos 2x$

$$= -\frac{1}{2}e^{-x}\cos 2x - \int \frac{1}{2}e^{-x}\cos 2x\,dx$$

$u = \frac{1}{2}e^{-x} \quad dv = \cos 2x\,dx$

$du = -\frac{1}{2}e^{-x}dx \quad v = \frac{1}{2}\sin 2x$

$$\int e^{-x}\sin(2x)\,dx = -\frac{1}{2}e^{-x}\cos 2x - \left(\frac{1}{4}e^{-x}\sin 2x - \int -\frac{1}{4}e^{-x}\sin 2x\,dx\right)$$

$$\frac{5}{4}\int e^{-x}\sin 2x\,dx = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{4}e^{-x}\sin 2x$$

$$\int e^{-x}\sin 2x\,dx = -\frac{2}{5}e^{-x}\cos 2x - \frac{1}{5}e^{-x}\sin 2x + C$$

11 pts

Pg 3 Tot: 22

(5) $\int \sqrt{1+e^x}\,dx$

$u=\sqrt{1+e^x}$, $du=\frac{1}{2\sqrt{1+e^x}}\cdot e^x\,dx \Rightarrow dx=\frac{2\sqrt{1+e^x}}{e^x}du=\frac{2u}{u^2-1}du$

$e^x=u^2-1$

$$=\int u\cdot\frac{2u}{u^2-1}du=\int\frac{2u^2}{u^2-1}du=\int 2+\frac{2}{u^2-1}du=\int 2+\frac{2}{(u-1)(u+1)}du$$

$$=\int 2+\frac{1}{u-1}+\frac{-1}{u+1}du=2u+\ln|u-1|-\ln|u+1|+C$$

$$=2\sqrt{1+e^x}+\ln\left|\sqrt{1+e^x}-1\right|-\ln\left|\sqrt{1+e^x}+1\right|+C$$

11 pts

(6) $\int \tan^{-1}x\,dx$

$u=\tan^{-1}x$, $dv=dx$

$du=\frac{1}{1+x^2}dx$ $\quad v=x$

$$=x\tan^{-1}x-\int\frac{x}{1+x^2}dx$$

$$=x\tan^{-1}x-\frac{1}{2}\ln(1+x^2)+C$$

11 pts

(7) $\int \frac{x+1}{x^2+4}\,dx = \int \frac{x}{x^2+4}+\frac{1}{x^2+4}\,dx$

$$=\frac{1}{2}\int\frac{2x}{x^2+4}dx+\frac{1}{4}\int\frac{1}{\left(\frac{x}{2}\right)^2+1}dx$$

$u=x^2+4$ $\quad u=\frac{x}{2}$ $\quad du=\frac{1}{2}dx$

$du=2x\,dx$

$$=\frac{1}{2}\int\frac{1}{u}du+\frac{1}{2}\int\frac{1}{u^2+1}du$$

$$=\frac{1}{2}\ln|u|+\frac{1}{2}\tan^{-1}u+C$$

$$=\frac{1}{2}\ln(x^2+4)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$$

11 pts

Pg 4 Tot: 33