Name : ______________________________

Exam 2 Honor's Calculus II March 3, 2006

I. Evaluate the following improper integrals. State whether the integral converges or diverges.

(1) $\displaystyle\int_{1/2}^{1} \frac{dx}{\sqrt{1-x^2}}$

$$= \lim_{b \to 1^-} \int_{1/2}^{b} \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{b \to 1^-} \sin^{-1}(b) - \sin^{-1}\left(\tfrac{1}{2}\right) = \sin^{-1}(1) - \sin^{-1}\left(\tfrac{1}{2}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

Convergent

12 pts

(2) $\displaystyle\int_{2}^{\infty} \frac{dx}{x\sqrt{\ln x}} = \lim_{b \to \infty} \int_{2}^{b} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$$= \lim_{b \to \infty} \int_{\ln 2}^{b} \frac{1}{\sqrt{u}}\,du$$

$$= \lim_{b \to \infty} 2\sqrt{u}\,\Big|_{\ln 2}^{b}$$

$$= \lim_{b \to \infty} 2\sqrt{b} - 2\sqrt{\ln 2}$$

∞

divergent

12 pts

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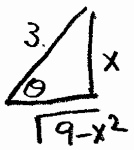
II. Evaluate the following integrals. Partial Credit will be given only for application of appropriate methods.

(1) $\displaystyle\int \frac{dx}{x\sqrt{9-x^2}}$



$x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$

$$\int \frac{3\cos\theta\, d\theta}{3\sin\theta\sqrt{9-9\sin^2\theta}} = \int \frac{\cancel{3\cos\theta}}{3\sin\theta \cdot \cancel{3\cos\theta}}\, d\theta$$

$$= \int \frac{1}{3}\csc\theta\, d\theta$$

$$= \frac{1}{3}\ln|\csc\theta - \cot\theta| + C$$

$$= \frac{1}{3}\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + C$$

12 pts

(2) $\displaystyle\int \frac{x^3}{\sqrt{x^2-1}}\,dx$

$u = x^2 - 1 \quad du = 2x\,dx \quad$ or $\quad x\,dx = \frac{1}{2}du \quad \& \quad x^2 = u+1$

$$\int \frac{x^2}{\sqrt{x^2-1}}\, x\,dx = \int \frac{u+1}{\sqrt{u}} \cdot \frac{1}{2}du = \frac{1}{2}\int u^{1/2} + u^{-1/2}\, du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) + C$$

$$= \frac{1}{3}(x^2-1)^{3/2} + \sqrt{x^2-1} + C$$

12 pts

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(3) $\int e^{\sqrt{x+3}}\,dx$

$t = \sqrt{x+3}$ or $x = t^2 - 3 \Rightarrow dx = 2t\,dt$

$$= \int 2t e^t \, dt$$

$u = 2t \quad dv = e^t dt$

$du = 2dt \quad v = e^t$

$$= 2te^t - \int 2e^t \, dt$$

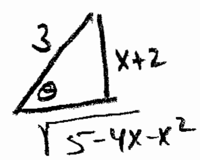
$$= 2te^t - 2e^t + C$$

$$= 2e^{\sqrt{x-3}}\left(\sqrt{x-3} - 1\right) + C$$

12 pts

(4) $\int \sqrt{5 - 4x - x^2}\, dx$

$$= \int \sqrt{9 - (x+2)^2}$$



$x + 2 = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$$= \int \sqrt{9 - 9\sin^2\theta} \cdot 3\cos\theta \, d\theta$$

$$= \int 9\cos^2\theta \, d\theta$$

$$= \int \frac{9}{2}(1 + \cos 2\theta)\, d\theta$$

$$= \frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x+2}{3}\right) + \frac{9}{2}\,\frac{x+2}{3} \cdot \frac{\sqrt{5-4x-x^2}}{3} + C$$

12 pts

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(5) $\int \frac{3x^2+2x+72}{x(x^2+36)}\,dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+36}\,dx = \int \frac{(A+B)x^2+Cx+A36}{x(x^2+36)}\,dx$

$A+B=3$ $\quad B=1$
$C=2$ $\quad C=2$
$36A=72 \Rightarrow A=2$

$= \int \frac{2}{x} + \frac{2x+1}{x^2+36}\,dx = \int \frac{2}{x} + \frac{2x}{x^2+36} + \frac{1}{x^2+36}\,dx$

$= 2\ln|x| + 2\ln(x^2+36) + \frac{1}{36}\int \frac{1}{(\frac{x}{6})^2+1}\,dx$

$u=\frac{x}{6} \quad dx = 6du$

$= 2\ln|x| + 2\ln(x^2+36) + \frac{1}{6}\tan^{-1}(\frac{x}{6}) + C$

12 pts

(6) $\int \frac{x^2-5x+8}{x(x-2)^2}\,dx = \int \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\,dx = \int \frac{A(x-2)^2+Bx(x-2)+Cx}{x(x-2)^2}\,dx$

$= \int \frac{A(x^2-4x+4)+B(x^2-2x)+Cx}{x(x-2)^2}\,dx = \int \frac{(A+B)x^2+(-4A-2B+C)x+4A}{x(x-2)^2}$

$A+B=1$ $\quad B=-1 \rightarrow -8+2+C=-5 \Rightarrow C=1$
$-4A-2B+C=-5$
$4A=8 \Rightarrow A=2$

$= \int \frac{2}{x} - \frac{1}{x-2} + \frac{1}{(x-2)^2}\,dx$

$= 2\ln|x| - \ln|x-2| - \frac{1}{x+2} + C$

12 pts

III. Find the general form of the following integral, without finding all necessary constants

(1) $\int \frac{6x^6-9x^5+17}{(x+1)^2(2x-1)(x^2+4x+13)}\,dx = \int \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1} + \frac{D(x+2)+E}{(x+2)^2+9}\,dx$

$= \int \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1} + D\frac{(x+2)}{(x+2)^2+9} + \frac{E}{9}\frac{1}{(\frac{x+2}{3})^2+1}\,dx$

6 pts

$= A\ln|x+1| - \frac{B}{x+1} + \frac{1}{2}C\ln|2x-1| + \frac{D}{2}\ln((x+2)^2+9) + \frac{E}{3}\tan^{-1}(\frac{x+2}{3}) +$ const