CALCULUS II - 3450:222
TEST #2 FALL 05

ROW # ____________

NAME ____

100 Points

Show all your work.

1. (8 pts) Evaluate $\int \ln(x)\,dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$= x\ln x - \int dx = x\ln x - x + C$

8 Points

2. (9 pts) Evaluate $\int \cos^5(x)\sin^3(x)\,dx$ $= \int \cos^5 x \sin^2 x \sin x\,dx$

$= \int \cos^5 x(1-\cos^2 x)\sin x\,dx \qquad u = \cos x, \; du = -\sin x\,dx$

$= -\int u^5(1-u^2)\,du$

$= -\int (u^5 - u^7)\,du$

$= -\left[\frac{1}{6}u^6 - \frac{1}{8}u^8\right] + C$

$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C$

9 Points

3. (8 pts) Use the Trapezoidal Rule to approximate to 5 decimal places $\int_1^2 e^{1/x}\,dx$ with n = 4.

$\Delta x = \frac{2-1}{4} = \frac{1}{4} \qquad 1, 5/4, 6/4, 7/4, 2$

$\int_1^2 e^{1/x}\,dx \approx \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left[f(1) + 2f(5/4) + 2f(3/2) + 2f(7/4) + f(2)\right]$

$= \frac{1}{8}\left[e + 2e^{4/5} + 2e^{2/3} + 2e^{4/7} + e^{1/2}\right]$

$= 2.03189$

8 Points

4. **(15 pts) Evaluate** $\int \frac{x+2}{x^4+x^2}dx$ $= \int \frac{x+2}{x^2(x^2+1)}\,dx$

$$\frac{x+2}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$x+2 = Ax(x^2+1) + B(x^2+1) + (Cx+D)(x^2)$$

let $x=0$, $2 = B$ $\Rightarrow$

$$x+2 = Ax^3 + Ax + 2x^2 + 2 + Cx^3 + Dx^2$$

$$x+2 = (A+C)x^3 + (D+2)x^2 + Ax + 2$$

coef. x: $A=1$ coef x^2: $D+2=0$, $D=-2$ coef x^3: $A+C=0$, $C=-1$

$$\int \frac{x+2}{x^4+x^2}dx = \int \frac{1}{x}dx + \int \frac{2}{x^2}dx + \int \frac{-x-2}{x^2+1}dx$$

$$= \ln|x| - 2x^{-1} - \int \frac{x}{x^2+1}dx - 2\int \frac{1}{x^2+1}dx$$

$u = x^2+1$, $du = 2x\,dx$

$$= \ln|x| - \frac{2}{x} - \frac{1}{2}\int \frac{1}{u}du - 2\tan^{-1}x + C$$

$$= \ln|x| - \frac{2}{x} - \frac{1}{2}\ln(x^2+1) - 2\tan^{-1}x + C$$

15 Points

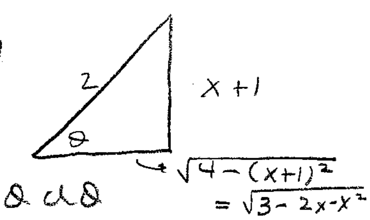
5. **(15 pts) Evaluate** $\int \sqrt{3-2x-x^2}\,dx$

$$= \int \sqrt{-(x^2+2x+1^2)+3-(-1)}\,dx = \int \sqrt{-(x+1)^2+4}\,dx$$

$$= \int \sqrt{4-(x+1)^2}\,dx$$

$x+1 = 2\sin\theta \Rightarrow \sin\theta = \frac{x+1}{2}$

$dx = 2\cos\theta\,d\theta$



$$= \int \sqrt{4-4\sin^2\theta}\;2\cos\theta\,d\theta = \int 2\cos\theta\;2\cos\theta\,d\theta$$

$$= 4\int \cos^2\theta\,d\theta = 2\int (1+\cos 2\theta)\,d\theta$$

$$= 2\left[\theta + \frac{1}{2}\sin 2\theta\right] + C = 2\theta + 2\sin\theta\cos\theta + C$$

$$= 2\sin^{-1}\left(\frac{x+1}{2}\right) + 2\left(\frac{x+1}{2}\right)\left(\frac{\sqrt{3-2x-x^2}}{2}\right) + C$$

15 Points

$$= 2\sin^{-1}\left(\frac{x+1}{2}\right) + \frac{(x+1)\sqrt{3-2x-x^2}}{2} + C$$

6. (11 pts) Evaluate $\int \sec^4(x)\sqrt{\tan(x)}\,dx$ $= \int \sec^2 x \sqrt{\tan x}\ \sec^2 x\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \int (1+\tan^2 x)\sqrt{\tan x}\ \sec^2 x\, dx$

$= \int (1+u^2)\, u^{1/2}\, du$

$= \int (u^{1/2} + u^{5/2})\, du$

$= \frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2} + C$

$= \frac{2}{3}\tan^{3/2} x + \frac{2}{7}\tan^{7/2} + C$

11 Points

7. (15 pts) Evaluate $\int e^x \cos(x)\,dx$

$u = e^x$ $\quad dV = \cos x\, dx$

$du = e^x\, dx$ $\quad V = \sin x$

$u = e^x$ $\quad dV = \sin x\, dx$

$du = e^x\, dx$ $\quad V = -\cos x$

$\int e^x \cos x\, dx = e^x \sin x - \int e^x \sin x\, dx$

$\int e^x \cos x\, dx = e^x \sin x - [-e^x \cos x + \int e^x \cos x\, dx]$

$\int e^x \cos x\, dx = e^x \sin x + e^x \cos x - \int e^x \cos x\, dx$

$2\int e^x \cos x\, dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x\, dx = \frac{1}{2}[e^x \sin x + e^x \cos x] + C$

15 Points

8. Determine whether the following integrals are convergent or divergent. Evaluate those that are convergent.

a. (10 pts) $\int_0^{\infty} \frac{1}{x^2 + 5x + 4} dx$

$= \int_0^{\infty} \frac{1}{(x+4)(x+1)} dx = \int_0^{\infty} \left(\frac{-1/3}{x+4} + \frac{1/3}{x+1} \right) dx$

$= \left[-\frac{1}{3} \ln|x+4| + \frac{1}{3} \ln|x+1| \right]_0^{\infty}$

$= \frac{1}{3} \left[\ln \left| \frac{x+1}{x+4} \right| \right]_0^{\infty}$

$= \frac{1}{3} \left[0 - \ln\left(\frac{1}{4}\right) \right]$

$= -\frac{1}{3} \ln\left(\frac{1}{4}\right)$ converges

10 Points

b. (9 pts) $\int_{-4}^{12} \frac{1}{(x-4)^{1/3}} dx = \int_{-4}^{4^-} \frac{1}{(x-4)^{1/3}} dx + \int_{4^+}^{12} \frac{1}{(x-4)^{1/3}} dx$

$= \frac{3}{2} (x-4)^{2/3} \Big|_{-4}^{4^-} + \frac{3}{2} (x-4)^{2/3} \Big|_{4^+}^{12}$

$= \frac{3}{2} [0 - 4] + \frac{3}{2} [4 - 0]$

$= -\frac{3}{2}(4) + \frac{3}{2}(4)$

$= 0$

converges

9 Points