Test Total

Name KEY

Test 1 Calculus II 3450:222 Dr. Clemons Feb. 10th, 2006

INSTRUCTIONS : Show all of your work.

1. Evaluate *and simplify* the derivatives of the following functions of x.

(a) (7 pts) $f(x) = (3x^2 - 1)4^x$

2 1

$f' = 6x\, 4^x + (3x^2-1)(\ln 4) \cdot 4^x$

1 1 1 1

(b) (7 pts) $g(x) = \ln \sqrt{e^{x^2}}$ $= \frac{1}{2}x^2$ 4

$g' = x$ 3

(c) (7 pts) $h(x) = x^2 \tan^{-1} x$

1

$h' = 2x \tan^{-1} x + \dfrac{x^2}{1+x^2}$ 3

1 1 1

(d) (7 pts) $j(x) = e^{-4x} \sin(3x)$

$j' = -4e^{-4x} \sin 3x + 3e^{-4x} \cos 3x$

1 each

(e) (7 pts) $k(x) = \log_3(1 + e^x)$ $= \dfrac{\ln(1+e^x)}{\ln 3}$ 3

$k' = \dfrac{1}{\ln 3 (1+e^x)} \cdot e^x$

2 2

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2. Evaluate and simplify the following integrals :

(a) (8 pts) $\int x \ln x \, dx$

$$\left.\begin{aligned} u &= \ln x, \quad dv = x\,dx \\ du &= \frac{1}{x}dx, \quad v = \frac{1}{2}x^2 \end{aligned}\right\} 2$$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2} x \, dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

1 1 1 1 1 1

(b) (8 pts) $\int_1^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}}$

$u = \ln x, \quad du = \frac{1}{x}dx$ 2

$$= \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}} = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1} 0$$

$$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

(c) (8 pts) $\int \frac{t-3}{t^2+16} \, dt$

$$= \int \frac{t}{t^2+16}\,dt - 3\int \frac{1}{t^2+16}\,dt$$

$$= \frac{1}{2}\int \frac{2t}{t^2+16}\,dt - \frac{3}{16}\int \frac{1}{\left(\frac{t}{4}\right)^2+1}\,dt$$ 1

$u = \frac{t}{4}, \quad dt = 4\,du$

$$= \frac{1}{2}\int \frac{2t}{t^2+16} - \frac{3}{4}\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{2}\ln(t^2+16) - \frac{3}{4}\tan^{-1}\left(\frac{t}{4}\right) + C$$

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(d) (8 pts) $\int (1+\tan^2 x)e^{\tan x}\,dx$

$$= \int \sec^2 x\, e^{\tan x}\, dx$$

$u = \tan x$, $du = \sec^2 x\, dx$ (4)

$$= \int e^u\, du = e^u + C = e^{\tan x} + C$$

(2) (1) (1)

3. Use l'Hopital's Rule to evaluate :

(a) (7 pts) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$

$$= \lim_{x\to 0} \frac{\sin x - x}{x \sin x} : \frac{0}{0} \quad (2)$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\cos x - 1}{\sin x + x\cos x} : \frac{0}{0} \quad (2)$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{-\sin x}{\cos x + \cos x - x\sin x} = \frac{0}{2} = 0 \quad (3)$$

(b) (6 pts) $\lim_{x\to \frac{\pi}{2}^-} (1+4\cot x)^{\tan x}$

$$e^{\lim_{x\to \frac{\pi}{2}^-} \tan x \ln(1+4\cot x)} \quad (2)$$

$$= e^{\lim_{x\to \frac{\pi}{2}^-} \frac{\ln(1+4\cot x)}{\cot x}} \quad \frac{0}{0} \quad (1)$$

$$\overset{L'H}{=} e^{\lim_{x\to \frac{\pi}{2}^-} \frac{\frac{1}{1+4\cot x}\cdot -4\cancel{\csc^2 x}}{-\cancel{\csc^2 x}}} \quad (2)$$

$$= e^{\frac{1}{1+4\cdot 0}\cdot 4} = e^4 \quad (1)$$

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4. Consider the function $f(x) = \cosh(e^x)$.

(a) (7 pts) Show that $f(x)$ has an inverse on its whole domain.

$$f' = \underset{+}{\sinh(e^x)} \cdot \underset{+}{e^x} > 0 \quad \text{So 1-1 \& has inverse}$$

since $e^x > 0$ 2 2 3

(b) (6 pts) Find the *slope* of the tangent line to $y = f^{-1}(x)$ at the point $\left(\frac{e^2 + e^{-2}}{2}, \ln 2\right)$.

$$f^{-1\prime}\left(\frac{e^2+e^{-2}}{2}\right) = \frac{1}{f'(\ln 2)} = \frac{1}{\sinh(e^x)\cdot e^x} = \frac{1}{\sinh(e^{\ln 2})\cdot e^{\ln 2}}$$

5 2

$$= \frac{1}{2\sinh 2}$$

2

5. (9 pts) Use logarithmic differentiation to find the derivative of

$f(x) = \cos^5 x \sqrt{\dfrac{x^2-1}{x^2+1}}$

1 $\ln f = 5 \ln \cos x + \frac{1}{2} \ln(x^2-1) - \frac{1}{2} \ln(x^2+1)$

1 1 2

2 $\frac{f'}{f} = \frac{5}{\cos x} \cdot -\sin x + \frac{1}{2} \cdot \frac{1}{x^2-1} \cdot 2x - \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x$

$f' = \left(-5\tan x + \frac{x}{x^2-1} - \frac{x}{x^2+1}\right) f$

$f' = \left(-5\tan x + \frac{x}{x^2-1} - \frac{x}{x^2+1}\right) \cdot \cos^5 x \sqrt{\frac{x^2-1}{x^2+1}}$

2

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