CALCULUS II - 3450:222
TEST #1 SPRING 03

NAME ______________________________

100 Points

Show all your work.

1. (6 pts) If g is the inverse function of $f(x) = x^5 + 3x^3 + 2x - 1$, find $g'(-1)$

$f: (0, -1)$ $\quad f'(x) = 5x^4 + 9x^2 + 2$

$g: (-1, 0)$ $\quad f'(0) = 2$

$$g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(0)} = \frac{1}{2}$$

6 Points

2. Given the one-to-one function $f(x) = \ln\left(\frac{1}{2}x + 3\right)$ answer the following questions:

a. (6 pts) Find $f^{-1}(x)$

$y = \ln(\frac{1}{2}x + 3)$

$x = \ln(\frac{1}{2}y + 3)$

$e^x = \frac{1}{2}y + 3$

$e^x - 3 = \frac{1}{2}y$

$f^{-1}(x) = y = 2(e^x - 3)$

b. (3 pts) What is the domain of f?

$\frac{1}{2}x + 3 > 0$ $\quad$ domain $= (-6, \infty)$

$\frac{1}{2}x > -3$

$x > -6$

c. (2 pts) What is the range of f?

range f = domain f^{-1}

range $f = \mathbb{R}$

11 Points

3. (4 pts) Use the properties of logarithms to simplify $\ln\left(\frac{e^{x^2}\sin x}{x}\right)$.

$$\ln\left(\frac{e^{x^2}\sin x}{x}\right) = \ln(e^{x^2}) + \ln(\sin x) - \ln(x)$$
$$= x^2 + \ln(\sin x) - \ln(x)$$

4 Points

4. Find the derivatives of the following functions: (You only need to simplify your answer in part e.)

a. (4 pts) $f(x) = 1.6^x + x^{1.6}$

$f'(x) = 1.6^x(\ln 1.6) + 1.6x^{0.6}$

b. (3 pts) $f(x) = e^{\sin(5x)}$

$f'(x) = e^{\sin(5x)}\cos(5x)(5)$

c. (6 pts) $f(x) = \sin^{-1}\left(\sqrt{\ln x - x^3}\right)$

$f'(x) = \dfrac{1}{\sqrt{1-(\ln x - x^3)}} \cdot \left(\frac{1}{2}\right)(\ln x - x^3)^{-1/2}$

$\cdot \left(\frac{1}{x} - 3x^2\right)$

d. (10 pts) $y = x^{\sin x}$

$\ln y = (\sin x)(\ln x)$

$\left(\frac{1}{y}\right)y' = (\sin x)\left(\frac{1}{x}\right) + (\ln x)(\cos x)$

$y' = y\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$

$y' = x^{\sin x}\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$

e. (8 pts) $f(x) = \dfrac{1 - \cosh x}{1 + \cosh x}$ (Be sure to simplify your answer.)

$$f'(x) = \frac{(1+\cosh x)(-\sinh x) - (1-\cosh x)(\sinh x)}{(1+\cosh x)^2}$$

$$= \frac{-\sinh x - \cosh x \sinh x - \sinh x + \cosh x \sinh x}{(1+\cosh x)^2}$$

$$= \frac{-2\sinh x}{(1+\cosh x)^2}$$

31 Points

5. Evaluate the following integrals:

a. (8 pts) $\int \left(\frac{\sin x}{4+\cos x}\right) dx$

let $u = 4 + \cos x$

$du = -\sin x \, dx$

$= -\int \frac{1}{u} du$

$= -\ln|u| + C$

$= -\ln|4+\cos x| + C$

$= -\ln(4+\cos x) + C$

b. (8 pts) $\int x e^{-x^2} dx$

let $u = -x^2$

$du = -2x \, dx$

$= -\frac{1}{2}\int e^u du$

$= -\frac{1}{2} e^u + C$

$= -\frac{1}{2} e^{-x^2} + C$

c. (10 pts) $\int \left(\frac{2x+1}{x^2+4}\right) dx$

$= \int \frac{2x}{x^2+4} dx + \int \frac{1}{x^2+4} dx$

let $u = x^2 + 4$

$du = 2x \, dx$

$= \int \frac{1}{u} du + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

$= \ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

26 Points

6. Evaluate the following limits:

a. (2 pts) $\lim_{x \to 2^+} e^{3/(2-x)}$

$= 0$

b. (3 pts) $\lim_{x \to \infty} \ln(10 + e^{-x^2})$

$= \ln(10 + 0)$

$= \ln(10)$

c. (7 pts) $\lim_{x \to 0} \left(\frac{e^x - 1 - x}{x^2} \right)$

$\overset{H}{=} \lim_{x \to 0} \left(\frac{e^x - 1}{2x} \right)$

$\overset{H}{=} \lim_{x \to 0} \left(\frac{e^x}{2} \right)$

$= \frac{1}{2}$

d. (10 pts) $\lim_{x \to 0} (1 - 2x)^{1/x}$

$\lim_{x \to 0} \left[\frac{1}{x} \ln(1 - 2x) \right]$

$= \lim_{x \to 0} \left[\frac{\ln(1-2x)}{x} \right]$

$\overset{H}{=} \lim_{x \to 0} \left[\frac{\left(\frac{1}{1-2x}\right)(-2)}{1} \right]$

$= \lim_{x \to 0} \left[\frac{-2}{1-2x} \right]$

$= -2$

$\lim_{x \to 0} (1 - 2x)^{1/x} = e^{-2}$

22 Points