

NAME _____

100 Points

Show all your work.

1. (6 pts) If g is the inverse function of $f(x) = x^5 + 3x^3 + 2x - 1$, find $g'(-1)$

$$f: (0, -1) \quad f'(x) = 5x^4 + 9x^2 + 2$$

$$g: (-1, 0) \quad f'(0) = 2$$

$$g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(0)} = \frac{1}{2}$$

6 Points

2. Given the one-to-one function $f(x) = \ln\left(\frac{1}{2}x + 3\right)$ answer the following questions:

- a. (6 pts) Find $f^{-1}(x)$

$$y = \ln\left(\frac{1}{2}x + 3\right)$$

$$x = \ln\left(\frac{1}{2}y + 3\right)$$

$$e^x = \frac{1}{2}y + 3$$

$$e^x - 3 = \frac{1}{2}y$$

$$f^{-1}(x) = y = 2(e^x - 3)$$

- b. (3 pts) What is the domain of f ?

$$\begin{aligned} \frac{1}{2}x + 3 &> 0 \\ \frac{1}{2}x &> -3 \\ x &> -6 \end{aligned}$$

$$\text{domain} = (-6, \infty)$$

- c. (2 pts) What is the range of f ?

$$\text{range } f = \text{domain } f^{-1}$$

$$\text{range } f = \mathbb{R}$$

11 Points

3. (4 pts) Use the properties of logarithms to simplify $\ln\left(\frac{e^{x^2} \sin x}{x}\right)$.

$$\begin{aligned} \ln\left(\frac{e^{x^2} \sin x}{x}\right) &= \ln(e^{x^2}) + \ln(\sin x) - \ln(x) \\ &= x^2 + \ln(\sin x) - \ln(x) \end{aligned}$$

4 Points

4. Find the derivatives of the following functions: (You only need to simplify your answer in part e.)

a. (4 pts) $f(x) = 1.6^x + x^{1.6}$

$$f'(x) = 1.6^x (\ln 1.6) + 1.6x^{0.6}$$

b. (3 pts) $f(x) = e^{\sin(5x)}$

$$f'(x) = e^{\sin(5x)} \cos(5x) (5)$$

c. (6 pts) $f(x) = \sin^{-1}(\sqrt{\ln x - x^3})$

$$f'(x) = \frac{1}{\sqrt{1-(\ln x - x^3)}} \cdot \left(\frac{1}{2}\right)(\ln x - x^3)^{-1/2} \cdot \left(\frac{1}{x} - 3x^2\right)$$

d. (10 pts) $y = x^{\sin x}$

$$\ln y = (\sin x)(\ln x)$$

$$\left(\frac{1}{y}\right)y' = (\sin x)\left(\frac{1}{x}\right) + (\ln x)(\cos x)$$

$$y' = y \left[\frac{\sin x}{x} + (\ln x)(\cos x) \right]$$

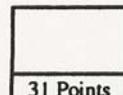
$$y' = x^{\sin x} \left[\frac{\sin x}{x} + (\ln x)(\cos x) \right]$$

e. (8 pts) $f(x) = \frac{1 - \cosh x}{1 + \cosh x}$ (Be sure to simplify your answer.)

$$f'(x) = \frac{(1 + \cosh x)(-\sinh x) - (1 - \cosh x)(\sinh x)}{(1 + \cosh x)^2}$$

$$= \frac{-\sinh x - \cosh x \sinh x - \sinh x + \cosh x \sinh x}{(1 + \cosh x)^2}$$

$$= \frac{-2 \sinh x}{(1 + \cosh x)^2}$$



5. Evaluate the following integrals:

a. (8 pts) $\int \left(\frac{\sin x}{4 + \cos x} \right) dx$

let $u = 4 + \cos x$
 $du = -\sin x dx$

$$= - \int \frac{1}{u} du$$

$$= -\ln|u| + C$$

$$= -\ln|4 + \cos x| + C$$

$$= -\ln(4 + \cos x) + C$$

b. (8 pts) $\int x e^{-x^2} dx$

let $u = -x^2$
 $du = -2x dx$

$$= -\frac{1}{2} \int e^u du$$

$$= -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

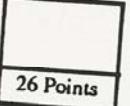
c. (10 pts) $\int \left(\frac{2x+1}{x^2+4} \right) dx$

$$= \int \frac{2x}{x^2+4} dx + \int \frac{1}{x^2+4} dx$$

let $u = x^2 + 4$
 $du = 2x dx$

$$= \int \frac{1}{u} du + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \ln(x^2 + 4) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$



6. Evaluate the following limits:

a. (2 pts) $\lim_{x \rightarrow 2^+} e^{3/(2-x)}$

= ∞

b. (3 pts) $\lim_{x \rightarrow \infty} \ln(10 + e^{-x^2})$

= $\ln(10 + 0)$

= $\ln(10)$

c. (7 pts) $\lim_{x \rightarrow 0} \left(\frac{e^x - 1 - x}{x^2} \right)$

$\stackrel{H}{=} \lim_{x \rightarrow 0} \left(\frac{e^x - 1}{2x} \right)$

$\stackrel{H}{=} \lim_{x \rightarrow 0} \left(\frac{e^x}{2} \right)$

= $\frac{1}{2}$

d. (10 pts) $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$

$\lim_{x \rightarrow 0} \left[\frac{1}{x} \ln(1 - 2x) \right]$

$= \lim_{x \rightarrow 0} \left[\frac{\ln(1 - 2x)}{x} \right]$

$\stackrel{H}{=} \lim_{x \rightarrow 0} \left[\frac{\frac{1}{1-2x}(-2)}{1} \right]$

$= \lim_{x \rightarrow 0} \left[\frac{-2}{1-2x} \right]$

= -2

$\lim_{x \rightarrow 0} (1 - 2x)^{1/x} = e^{-2}$

