Honors CALC II Test IC Grader: Stroup

① differentiate

a) $f(x) = e^{\ln(x^2+x)} = x^2+x$

$f'(x) = 2x+1$

b) $g(x) = \sin^{-1}(e^{2x}) \Rightarrow g'(x) = \dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$

$$\boxed{\begin{array}{ll} u = e^{2x} & g = \sin^{-1}(u) \\ du = 2e^{2x}dx & dg = \dfrac{du}{\sqrt{1-u^2}} \end{array}}$$

c) $h(x) = \sec^{-1}(\cos x) \Rightarrow h'(x) = \dfrac{-\sin x}{\cos x\sqrt{\cos^2 x-1}}$

$$\boxed{\begin{array}{ll} u = \cos x & h = \sec^{-1}(u) \\ du = -\sin x\,dx & dh = \dfrac{1}{u\sqrt{u^2-1}} \end{array}}$$

d) $j(x) = e^{2x}\sin(3x)$

$j'(x) = 3e^{2x}\cos(3x) + 2e^{2x}\sin(3x)$

e) $k = \log_5\left(\sqrt[3]{x^2}\right) = \dfrac{\ln(x^{2/3})}{\ln 5} = \dfrac{2\ln x}{3\ln 5}$

$k' = \dfrac{2}{3x\ln 5}$

② integrate

a) $\int 5^{2x+3}\,dx = \int e^{(2x+3)\ln 5}\,dx = \frac{1}{2\ln 5}\int e^u\,du$

$\boxed{\begin{aligned} u &= (2x+3)\ln 5 \\ du &= 2\ln 5\,dx \\ dx &= \frac{du}{2\ln 5} \end{aligned}}$

$= \frac{e^u}{2\ln 5} + C = \frac{5^{2x+3}}{2\ln 5}$

b) $\int \frac{dx}{x\ln x\sqrt{(\ln x)^2-1}} = \int \frac{du}{u\sqrt{u^2-1}}$

$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{dx}{x} \\ dx &= x\,du \end{aligned}}$

$= \sec^{-1}(u) + C$

$= \sec^{-1}(\ln x) + C$

c) $\int \frac{x+2}{x^2+4}\,dx = \int \frac{x}{x^2+4}\,dx + \int \frac{2\,dx}{x^2+4}$

$\boxed{\begin{aligned} u &= x^2+4 \\ du &= 2x\,dx \\ dx &= \frac{du}{2x} \end{aligned}}$

$= \frac{1}{2}\int \frac{du}{u} + \int \frac{2\,dx}{4\left(\left(\frac{x}{2}\right)^2+1\right)}$

$= \frac{1}{2}\int \frac{du}{u} + \int \frac{dz}{z^2+1}$

$\boxed{\begin{aligned} z &= \frac{x}{2} \\ dz &= \frac{dx}{2} \\ dx &= 2\,dz \end{aligned}}$

$= \frac{1}{2}\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C$

2e) $\int \frac{\tan x}{\cos^3 x}\,dx = \int \frac{\sin x}{\cos^4 x}\,dx$ let $u = \cos x$, $du = -\sin x\,dx$

$$= \int -\frac{1}{u^4}\,du = \frac{1}{3}\cdot\frac{1}{u^3} + C = \frac{1}{3}\cdot\frac{1}{\cos^3 x} + C$$

2f) $\int \tan^{-1} x\,dx$

let $u = \tan^{-1} x$ $\quad dv = dx$

$du = \frac{1}{1+x^2}$ $\quad v = x$

$$= x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$$

2g) $\int \frac{1}{\sqrt{-x^2-4x-3}}\,dx = \int \frac{1}{\sqrt{1-(x+2)^2}}\,dx = \sin^{-1}(x+2) + C$

$$\int \frac{\operatorname{sech}^2(1+\ln x)\,dx}{x} = \int \operatorname{sech}^2(u)\,du$$

$$\boxed{\begin{aligned} u &= 1+\ln x \\ du &= \frac{dx}{x} \\ dx &= x\,du \end{aligned}}$$

$$= \tan u + C$$

$$= \tan(1+\ln x) + C$$

3) find limit

a) See Exam 1B #3b

b) $$\lim_{x\to 1^+} \frac{\ln x - x + 1}{x^2 - 3x + 2} \overset{L'H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x} - 1}{3x^2 - 3}$$

$$\overset{L'H}{=} \lim_{x\to 1^+} \frac{\frac{-1}{x^2}}{6x} = \frac{-1}{6}$$

4) given $f(x) = \sinh(2x) + 5e^{3x}$

a) show $f(x)$ has inverse on whole domain

$f'(x) = 2\cosh(2x) + 15e^{3x} \Rightarrow y > 0$ for all x

$\Rightarrow y$ is 1-1

$\Rightarrow y$ has inverse on whol

b) $$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(0)} = \frac{1}{2+15} = \frac{1}{17}$$

5 $y = (x^2+x)^5 \dfrac{\sqrt[3]{2-3x}}{\sqrt[4]{x+1}}$

$\ln y = 5\ln(x^2+x) + \frac{1}{3}\ln(2-3x) - \frac{1}{4}\ln(x+1)$

$y' = (x^2+x)^5 \dfrac{\sqrt[3]{2-3x}}{\sqrt[4]{x+1}} \left[\dfrac{10x+5}{x^2+x} + \dfrac{1}{3(2-3x)} - \dfrac{1}{4(x+1)} \right]$