Honors CALC II Test 1B Grader: Stroup (A) 2/7

① differentiate

a) $f(x) = x^{\sin x} = e^{\sin x \ln x}$

$f'(x) = x^{\sin x}\left[\cos x \ln x + \frac{\sin x}{x}\right]$

b) $g(x) = (3x^2 - x + 1)e^{5x}$

$= (6x-1)e^{5x} + (3x^2 - x + 1)(5e^{5x})$

c) $h(x) = \sin^{-1}(\sqrt{x}) \Rightarrow h'(x) = \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{1-x}}$

$$\boxed{\begin{array}{ll} u = \sqrt{x} & h = \sin^{-1}(u) \\ du = \frac{dx}{2\sqrt{x}} & dh = \frac{du}{\sqrt{1-u^2}} \end{array}}$$

d) $j(x) = e^{-5x}\cos(2x)$

$j'(x) = -2e^{-5x}\sin(2x) - 5e^{-5x}\cos(2x)$

e) $k(x) = \log_2(x) = \frac{\ln x}{\ln 2}$

$k'(x) = \frac{1}{x \ln 2}$

) integrate

a) $\int \frac{dx}{x\sqrt{\ln(x)}} = \int \frac{du}{\sqrt{u}} = 2u^{\frac{1}{2}} + C$

$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{dx}{x} \\ dx = x\,du \end{array}}$

$= 2(\ln x)^{\frac{1}{2}} + C$

b) $\int_1^e \frac{dx}{x\sqrt{5-2\ln x}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -u^{\frac{1}{2}}\Big|_{u(1)}^{u(e)}$

$\boxed{\begin{array}{l} u = 5-2\ln x \\ du = \frac{-2dx}{x} \\ dx = \frac{-x\,du}{2} \end{array}}$

$= -(5-2\ln x)^{\frac{1}{2}}\Big|_1^e$

$= -(5-2\ln e)^{\frac{1}{2}} + \sqrt{5} = -\sqrt{3} + \sqrt{5}$

c) $\int \frac{t+3}{4t^2+9}\,dt = \int \frac{t\,dt}{4t^2+9} + \int \frac{3\,dt}{4t^2+9} = \frac{1}{8}\int \frac{du}{u} + \frac{3}{9}\int \frac{d}{(\frac{2}{3}t}$

$\boxed{\begin{array}{l} u = 4t^2+9 \\ du = 8t\,dt \\ dt = \frac{du}{8t} \end{array}}$

$= \frac{1}{8}\int \frac{du}{u} + \frac{1}{3}\cdot\frac{3}{2}\int \frac{dz}{z^2+1}$

$= \frac{1}{8}\ln(4t^2+9) + \frac{1}{2}\tan^{-1}\left(\frac{2}{3}t\right) +$

$\boxed{\begin{array}{l} z = \frac{2}{3}t \\ dz = \frac{2}{3}dt \\ dt = \frac{3}{2}dz \end{array}}$

$\int (x+1)\ln x\,dx$

$u = \ln x \quad dv = x+1\,dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2+x$

$= (\frac{1}{2}x^2+x)\ln x - \int (\frac{1}{2}x^2+x)\cdot\frac{1}{x}\,dx$

$= (\frac{1}{2}x^2+x)\ln x - \frac{1}{4}x^2 - x + C$ $= (\frac{1}{2}x^2+x)\ln x - \int \frac{1}{2}x+1\,dx$

2 f)

$\int \sec x \tan^3 x\,dx = \int \tan^2 x \cdot \sec x \tan x\,dx$

$= \int (\sec^2 x - 1)\cdot \sec x \tan x\,dx$

$u = \sec x \quad du = \sec x \tan x\,dx$

$= \int u^2 - 1\,du = \frac{1}{3}u^3 - u + C = \frac{1}{3}(\sec x)^3 - (\sec x) + C$

2g)

$\int e^{\cos x}\sin^3 x\,dx = \int e^{\cos x}\sin^2 x \cdot \sin x\,dx$

$= \int e^{\cos x}(1-\cos^2 x)\cdot \sin x\,dx$

Let $t = \cos x \quad dt = -\sin x\,dx$

$= \int (t^2-1)e^t\,dt$

Let $u = (t^2-1),\ dv = e^t\,dt$

$du = 2t\,dt,\ v = e^t$

$= (t^2-1)e^t - \int 2te^t\,dt$

Let $u = 2t,\ dv = e^t\,dt$

$du = 2\,dt,\ v = e^t$

$= (t^2-1)e^t - (2te^t - \int 2e^t\,dt)$

$= (t^2-1)e^t - 2te^t + 2e^t + C$

$= (t^2 - 2t + 1)e^t + C$

$= (\cos^2 x - 2\cos x + 1)e^{\cos x} + C$

d) $\int e^{2t}\sinh(1+e^{2t}) = \frac{1}{2}\int \sinh u\,du$

$$\boxed{\begin{aligned} u &= 1+e^{2t} \\ du &= 2e^{2t}\,dt \\ dt &= \frac{du}{2e^{2t}} \end{aligned}}$$

$= \frac{1}{2}\cosh u + C$

$= \frac{1}{2}\cosh(1+e^{2t}) + C$

3) find limit

a) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right) = \lim_{x\to 0}\frac{e^x-1-x}{xe^x-x} \overset{L'H}{=} \lim_{x\to 0}\frac{e^x-1}{e^x+xe^x-1}$

$\overset{L'H}{=} \lim_{x\to 0}\frac{e^x}{e^x+e^x+xe^x} = \lim_{x\to 0}\frac{1}{2+x} = \frac{1}{2}$

b) $\lim_{x\to 0^+}|\ln x|^x$, let $y = |\ln x|^x \to \ln y = x\ln|\ln x|$

$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+} x\ln|\ln x| = \lim_{x\to 0^+}\frac{\ln|\ln x|}{1/x} \overset{L'H}{=} \lim_{x\to 0^+}\frac{\frac{abs(1,\ln x)}{x|\ln x|}}{-1/x^2}$

$= \lim_{x\to 0^+}\frac{-x\,abs(1,\ln x)}{|\ln x|} \overset{L'H}{=} \lim_{x\to 0^+}\frac{-abs(1,\ln x)}{\frac{abs(1,\ln x)}{x}} - \frac{x\,sgn(1,\ln x)}{\frac{abs(1,\ln x)}{x}}$

$= \lim_{x\to 0^+} -x - \frac{x^2\,sgn(1,\ln x)}{abs(1,\ln x)} = 0$

$\Rightarrow \lim_{x\to 0^+}|\ln x|^x = \lim_{x\to 0^+} e^{x\ln|\ln x|} = e^0 = 1$

Note: $\lim_{x\to 0^+} abs(1,\ln x) = -1$, $\lim_{x\to 0^+} sgn(1,\ln x) = 0$

abs(1, lnx) and sgn(1, lnt) are maple notation → look it up!

(4) given $f(x) = 5x + \sin 2x$

a) prove $f(x)$ has inverse on its whole domain

$y' = 5 + 2\cos 2x \Rightarrow y' > 0$ for all x

$\Rightarrow y$ is 1-1

$\Rightarrow y$ has inverse on its domain

b) $$g'\left(\frac{5\pi}{2}\right) = \frac{1}{f'\left(g\left(\frac{5\pi}{2}\right)\right)} = \frac{1}{f'\left(\frac{\pi}{2}\right)} = \frac{1}{5+2\cos(\pi)} = \frac{1}{3}$$

(5) $$y = (\sin x)^3 \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}$$

$$\ln y = 3\ln \sin x + \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$$

$$y' = (\sin x)^3 \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} \left[\frac{3\cos x}{\sin x} + \frac{1}{3(x-1)} + \frac{1}{3(x+1)}\right]$$