Honors Calculus II: Spring 2007
Exam 2

Name:____

1.[60pts.] Evaluate each of the following integrals. Each is worth 10 points.

(a) $\int \tan x \sin 2x \, dx.$

$$= \int \tan x \cdot 2 \sin x \cos x \, dx \quad 3$$

$$= \int 2 \sin^2 x \, dx \quad 1$$

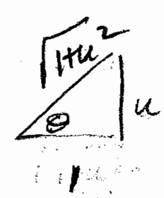
$$= \int 1 - \cos 2x \, dx \quad 3$$

$$= x - \tfrac{1}{2} \sin 2x + C$$

1 1 1

(b) $\int \frac{1}{(x+3)\sqrt{(x+3)^2+1}} \, dx.$

$u = x+3$



$$\int \frac{1}{u\sqrt{u^2+1}} \, du$$

$u = \tan\theta \quad du = \sec^2\theta \, d\theta \quad 3$

$$\int \frac{1}{\tan\theta \sqrt{\tan^2\theta + 1}} \cdot \sec^2\theta \, d\theta$$

3 2

$$\int \frac{\sec\theta}{\tan\theta} d\theta = \int \csc\theta \, d = \ln|\csc\theta - \cot\theta| + C$$

$$= \ln\left|\frac{\sqrt{1+u^2}}{u} - \frac{1}{u}\right| + C = \ln\left|\frac{\sqrt{1+(x+3)^2}}{x+3} - \frac{1}{x+3}\right| + C$$

2

(c) $\int_0^1 \frac{1}{1+\sqrt[3]{x}}\,dx.$

$u = x^{1/3}$, $x = u^3$, $dx = 3u^2\,du$ (3)

$\int_0^1 \frac{3u^2}{1+u}\,du = \int_0^1 3u - 3 + \frac{3}{1+u}\,du$ (3)

$= \frac{3}{2}u^2 - 3u + 3\ln(1+u) \Big|_0^1$ (1) (1) (1)

$= \frac{3}{2} - 3 + 3\ln 2$ (1)

(d) $\int \frac{3x-4}{x^2-4x+4}\,dx.$

$= \int \frac{3x-4}{(x-2)^2}\,dx$ (1)

$u = x-2$, $du = dx$

$x = u+2$

$= \int \frac{3u+6-4}{u^2}\,du$ (3)

$= \int \frac{3}{u} + \frac{2}{u^2}\,du$ (2)

$= 3\ln|u| - \frac{2}{u} + C$ (3)

$= 3\ln|x-2| - \frac{2}{x-2} + C$ (1)

(e) $\displaystyle\int \frac{x^3-4x^2-15x-22}{x^2-4x-21}\,dx = \int \frac{x^3-4x^2-15x-22}{(x-7)(x+3)}\,dx.$

$$\begin{array}{r|l} & x \\ x^2-4x-21 & x^3-4x^2-15x-22 \\ & -(x^3-4x^2-21x) \\ \hline & 6x-22 \end{array}$$

$$= \int x + \frac{6x-22}{(x-7)(x+3)}\,dx \quad 3$$

$$= \int x + \frac{2}{x-7} + \frac{4}{x+3}\,dx \quad 3$$ Handcover method

$$= \frac{1}{2}x^2 + 2\ln|x-7| + 4\ln|x+3| + C$$

1 1 1 1

(f) $\displaystyle\int \frac{x+2}{x^2-10x+26}\,dx.$

$$= \int \frac{x+2}{(x-5)^2+1}\,du \quad 2$$

$u = x-5$, $du = dx$, $x = u+5$

$$= \int \frac{u+7}{u^2+1}\,du \quad 2$$

$$= \frac{1}{2}\int \frac{2u}{u^2+1}\,du + 7\int \frac{1}{u^2+1}\,du \quad 2$$

1 1 1

$$= \frac{1}{2}\ln(u^2+1) + 7\tan^{-1}u + C$$

$$= \frac{1}{2}\ln\left((x-5)^2+1\right) + 7\tan^{-1}(x-5) + C \quad 1$$

2.[10pts.] Do one of the following. ** DO JUST ONE **

$$\int \frac{2x}{\sqrt{6-x^4}}\,dx. \qquad \text{or} \qquad \int \sec^2 x \ln(\tan x)\,dx.$$

$u = x^2 \quad du = 2x\,dx$

$= \int \frac{1}{\sqrt{6-u^2}}\,du$ 1

$= \frac{1}{\sqrt{6}} \int \frac{1}{\sqrt{1-\left(\frac{u}{\sqrt{6}}\right)^2}}\,du$ 2

$v = \frac{u}{\sqrt{6}} \cdot du = \sqrt{6}\,dv$ 2

$= \int \frac{1}{\sqrt{1-v^2}}\,dv$ 1

$= \sin^{-1} v + C$ 2

$= \sin^{-1} \frac{u}{\sqrt{6}} + C$

$= \sin^{-1} \frac{x^2}{\sqrt{6}} + C$ 2

$t = \tan x$

$dt = \sec^2 x\,dx$

$\int \ln t\,dt$ 2

$u = \ln t \quad dv = dt$ 2

$du = \frac{1}{t}dt \quad v = t$

$= t\ln t - \int t \cdot \frac{1}{t}\,dt$ 2

$= t(\ln t - 1) + C$ 2

$= \tan x(\ln(\tan x) - 1) + C$

2

3.[10pts.] Write out the correct form of the partial fraction decomposition of the expression

$$\frac{x^2}{(x-2)^2(x^2-x+5)^3}.$$

Do ** NOT ** find the numerical values of the constants.

$$\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2-x+5} + \frac{Ex+F}{(x^2-x+5)^2} + \frac{Gx+H}{(x^2-x+5)^3}$$

2 2 2 2 2

4.[20pts.] Consider $\int_0^1 e^x\,dx$. (a) Write down the expression you would evaluate to approximate this integral using the Trapezoidal rule with $n = 4$. (b) How large would you choose n to ensure that the approximation T_n is accurate to within .00001.

2 3

a) $\dfrac{\frac{1-0}{4}}{2}\left(e^0 + 2e^{\frac{1}{4}} + 2e^{1/2} + 2e^{3/4} + e^1\right)$

2 3

b) $E_n^T \leq \dfrac{(b-a)^3 M}{12n^2} = \dfrac{(1-0)^3 e^1}{12n^2} < .00001$

3 4

$n \geq \sqrt{\dfrac{e}{12(.00001)}} = 150.5$

1

Choose $n = 151$. 2