Honors Calculus II: Spring 2007
Exam 1

Name: KEY

1.[42pts.] Evaluate each of the following.

(a) $\displaystyle\int \frac{1}{\sqrt{x}(1+x)}\,dx$

$= \displaystyle\int \frac{1}{\sqrt{x}(1+(\sqrt{x})^2)}\,dx$

$u=\sqrt{x} \quad du = \frac{1}{2\sqrt{x}}dx$

$= \displaystyle\int \frac{2}{1+u^2}\,du$

$= 2\tan^{-1}u + C$

$= 2\tan^{-1}\sqrt{x} + C$

(b) $\displaystyle\int \frac{\cosh x}{\cosh^2 x - 1}\,dx$

$= \displaystyle\int \frac{\cosh x}{\sinh^2 x}\,dx \qquad \cosh^2 x - \sinh^2 x = 1$

$u = \sinh x \quad du = \cosh x\, dx$

$= \displaystyle\int \frac{1}{u^2}\,du$

$= -\frac{1}{u} + C$

$= \frac{-1}{\sinh x} + C$

$= -\operatorname{csch} x + C$

(c) $\displaystyle\int_1^e \frac{1}{x(3-5\ln x)}\,dx$

$u = 3 - 5\ln x$

$du = -\frac{5}{x}dx$ or $\frac{1}{x}dx = -\frac{1}{5}du$

$\displaystyle\int_3^{-2} -\frac{1}{5}\cdot\frac{1}{u}\,du$ ← note - integrand is not cont'n so the definite integral is not defined

$= -\frac{1}{5}\ln u + C$

$= -\frac{1}{5}\ln|3-5\ln x| + C$

(d) $\displaystyle\int p^5 \ln p\, dp$

$u = \ln p \quad dv = p^5\,dp$

$du = \frac{1}{p}dp \quad v = \frac{1}{6}p^6$

$= \frac{1}{6}p^6\ln p - \displaystyle\int \frac{1}{6}p^6\cdot\frac{1}{p}\,dp$

$= \frac{1}{6}p^6\ln p - \frac{1}{6}\displaystyle\int p^5\,dp$

$= \frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C$

(Problem 1 continued)

(e) $\int \frac{1}{\sqrt{1-16t^2}}\,dt$

$= \int \frac{1}{\sqrt{1-(4t)^2}}\,dt$

$u = 4t \quad dt = \frac{1}{4}du$

$= \int \frac{1}{4} \frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{4}\sin^{-1}u + C$

$= \frac{1}{4}\sin^{-1}(4t) + C$

(f) $\int 3^{\sin\theta}\cos\theta\,d\theta$

$u = \sin\theta \quad du = \cos\theta\,d\theta$

$= \int 3^u\,du$

$= \int e^{u \cdot \ln 3}\,du$

$= \frac{1}{\ln 3} e^{u \cdot \ln 3} + C$

$= \frac{1}{\ln 3} 3^u + C$

$= \frac{1}{\ln 3} \cdot 3^{\sin\theta} + C$

2.[20pts.] For each of the following, find $\frac{dy}{dx}$.

(a) $y = x^{e^x}$

$\ln y = e^x \ln x$

$\frac{y'}{y} = e^x \ln x + \frac{e^x}{x}$

$y' = x^{e^x}\left(e^x \ln x + \frac{e^x}{x}\right)$

(b) $y = \tan^{-1}(\ln(x-5))$

$y' = \frac{1}{1+\ln^2(x-5)} \cdot \frac{1}{x-5}$

(c) $y = \log_5(x-18)^{1/3}$

$y = \frac{1}{3\ln 5} \cdot \ln(x-18)$

$y' = \frac{1}{3\ln 5 \cdot (x-18)}$

(d) $y = \cosh^2(3x)$

$y' = 2\cosh(3x) \cdot \sinh(3x) \cdot 3$

3.[15pts.] Find the following limits.

(a) $\lim_{x\to 0}(1-2x)^{2/x}$ (b) $\lim_{x\to\infty}\frac{\ln(\ln x)}{x}$ (c) $\lim_{x\to 0}\frac{\sinh x}{\cosh x}$

$= e^{\lim_{x\to 0} \frac{2\ln(1-2x)}{x}}, \frac{0}{0}$

$\overset{L'H}{=} e^{\lim_{x\to 0} \frac{2\cdot\frac{1}{1-2x}\cdot -2}{1}}$

$= e^{\lim_{x\to 0} \frac{-4}{1-2x}}$

$= e^{-4}$

L'H

$= \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1}$

$= 0$

$= \frac{0}{1} = 0$

4.[8pts.] Solve each of the following for x.

(a) $\ln(2x) = 2 - 3\ln x$

$\ln(2x) + 3\ln x = 2$

$\ln(2x) + \ln x^3 = 2$

$\ln(2x^4) = 2$

$2x^4 = e^2$

$x = \sqrt[4]{\frac{e^2}{2}}$

(b) $e^{1/(x-5)} = 10$

$\ln e^{1/(x-5)} = \ln 10$

$\frac{1}{x-5} = \ln 10$

$x - 5 = \frac{1}{\ln 10}$

$x = 5 + \frac{1}{\ln 10}$

5.[15pts.] Consider the function $f(x) = \frac{1}{\sqrt{x^2+1}}$ with the domain $x < 0$. (a) Show that f is 1-1. (b) Find a formula for f^{-1} and state the domain and range of f^{-1}.

a) $f' = -\frac{1}{2}\cdot\frac{1}{(x^2+1)^{3/2}}\cdot 2x > 0$ for $x<0$ So inc. & 1-1.

NOTE: dom f: $x > 0$, range f: $(0,1)$

b) $y = \frac{1}{\sqrt{x^2+1}}$

$x = \frac{1}{\sqrt{y^2+1}}$

$\sqrt{y^2+1} = \frac{1}{x}$

$y^2+1 = \frac{1}{x^2}$

$y = +\sqrt{\frac{1}{x^2}-1} = f^{-1}$

dom f^{-1}: $(0,1)$

range f^{-1}: $y<0$.