$\bar{x} = 80$ min = 48
$\sigma = 12.5$ med = 84
$n = 26$ max = 99

Test Total

Name ______________________________

Test 3 Nov. 21st, 2007 Honors Calculus I 3450:221 Drs. Clemons/Norfolk
Show all of your work, and simplify your answers.

1. Use Part I of the Fundamental Theorem of Calculus to find the derivative of $y = \displaystyle\int_{\sqrt{x}}^{4} \frac{1}{\sqrt{t^2+5}}\,dt$

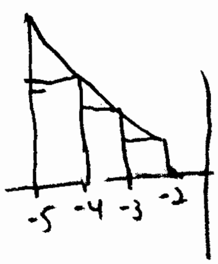
$$\left[\int_{\sqrt{x}}^{4} \frac{1}{\sqrt{t^2+5}}\,dt\right]' = -\left[\int_{4}^{\sqrt{x}} \frac{1}{\sqrt{t^2+5}}\,dt\right]$$

$$= \frac{-1}{\sqrt{(\sqrt{x})^2+5}} \cdot \frac{1}{2\sqrt{x}}$$

5 points

2. Estimate the area under the curve of $f(x) = 2x^2 + x$ from $x = -5$ to $x = -2$ using 3 sub-intervals and the right endpoints.

$x(2x+1)$



$$\Delta x = \frac{-2 - -5}{3} = \frac{3}{3} = 1$$

$$\Delta x\left[f(-4) + f(-3) + f(-2)\right] = 1\left[2(-4)^2 - 4 + 2(-3)^2 - 3 + 2(-2)^2 - 2\right]$$

$$= (6 + 15 + 28) = 49$$

7 points

3. A particle moves on the x-axis with *acceleration* $a(t) = \cos(t) - 30t\ m/s^2$. If the *velocity* at time $t = 0$ seconds is $v(0) = 3\ m/s$, and the position at time $t = 0$ seconds is $s(0) = \frac{\pi}{2}\ m$, find the position at time t.

$a = \cos(t) - 30t$

$V = \sin(t) - 15t^2 + C_1 \qquad v(0) = 3 = 0 - 0 + C_1 \Rightarrow C_1 = 3$

$S = -\cos(t) - 5t^3 + C_1 t + C_2 \qquad s(0) = \frac{\pi}{2} = -1 + C_2 \Rightarrow C_2 = 1 + \frac{\pi}{2}$

9 points

Page 1 Total (21 pts)

$S = -\cos t - 5t^3 + 3t + \left(1 + \frac{\pi}{2}\right)$

4. Sketch the graph of the function $f(x)$ that has the following properties:

$f(-2) = 0$; $f(0) = 3$; $f(1) = 2$; $f(2) = 1$;

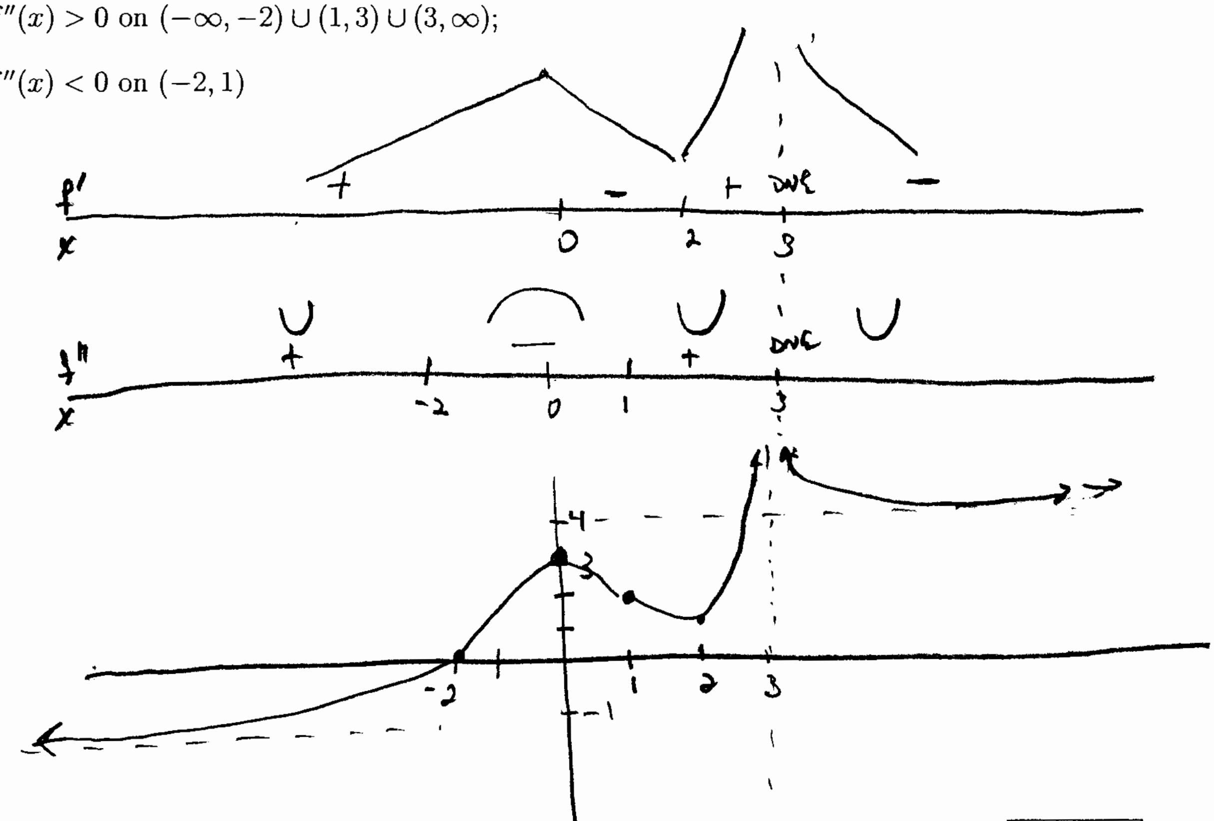
$\lim_{x \to -\infty} f(x) = -1$; $\lim_{x \to \infty} f(x) = 4$; $\lim_{x \to 3} f(x) = +\infty$;

$f'(x) > 0$ on $(-\infty, 0) \cup (2, 3)$;

$f'(x) < 0$ on $(0, 2) \cup (3, \infty)$;

$f''(x) > 0$ on $(-\infty, -2) \cup (1, 3) \cup (3, \infty)$;

$f''(x) < 0$ on $(-2, 1)$

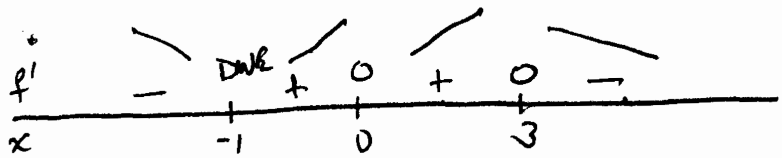


9 points

Page 2 Total(9 pts)

5. Given $f'(x) = \dfrac{x^2(3-x)}{1+x}$ and $f''(x) = \dfrac{2x(3-x^2)}{(1+x)^2}$, find the following :

(a) The intervals where $f(x)$ is increasing, and those where it is decreasing.



inc $(-1,0),(0,3)$

dec $(-\infty,-1),(3,\infty)$

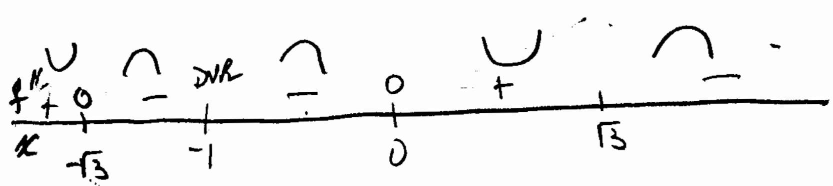
6 points

(b) Find the x-coordinates of any local extrema, and identify whether they are maxima or minima.

local max @ $x=3$

3 points

(c) Find the intervals on which $f(x)$ is concave up, and those where it is concave down.



C.U. $(-\infty,-\sqrt{3}),(0,\sqrt{3})$

CD $(-\sqrt{3},-1),(-1,0),(\sqrt{3},\infty$

6 points

(d) Find the x-coordinates of any points of inflection.
Note : $f(x)$ is *not* continuous.

inf pts @ $x=-\sqrt{3}, 0, \sqrt{3}$

3 points

Page 3 Total (18 pts)

6. Evaluate the following limits :

(a) $\lim_{x\to-\infty} \frac{5x^2-2x+11}{x-\sqrt{36x^4+9x^3}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to-\infty} \frac{5-\frac{2}{x}+\frac{11}{x^2}}{\frac{1}{x}-\sqrt{36+\frac{9}{x}}} = \frac{5}{-6}$

5 points

(b) $\lim_{t\to\infty} \left(4t-1-\sqrt{16t^2-8t}\right) * \frac{(4t-1+\sqrt{16t^2-8t})}{(4t-1+\sqrt{16t^2-8t})}$

$= \lim_{t\to\infty} \frac{(4t-1)^2-(16t^2-8t)}{4t+1+\sqrt{16t^2-8t}} = \lim_{t\to\infty} \frac{16t^2-8t+1-16t^2+8t}{4t+1+\sqrt{16t^2-8t}}$

$= \lim_{t\to\infty} \frac{1}{4t+1+\sqrt{16t^2-8t}} = 0$

10 points

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[3\left(1+\frac{i}{n}\right)^2+1\right] \cdot \frac{1}{n}$

$a+\Delta x i = 1+\frac{i}{n}$ $\qquad \Delta x = \frac{1}{n} = \frac{b-a}{n}, \quad a=1 \Rightarrow b=2$

$\int_1^2 3x^2+1\,dx$

$= x^3+x \Big|_1^2 = (8+2)-(1+1)$

$= 8$

10 points

Page 4 Total (25 pts)

7. Evaluate the following :

(a) $\int_1^4 \frac{x^3-1}{\sqrt{x}}\,dx$ $= \int_1^4 x^{5/2} - x^{-1/2}\,dx$

$= \frac{2}{7}x^{7/2} - 2x^{1/2} \Big|_1^4$

$= \left(\frac{2}{7}4^{7/2} - 2\sqrt{4}\right) - \left(\frac{2}{7} - 2\right)$

$= \frac{2}{7}(4^{7/2}) - 2 = 34.57 = \frac{242}{7}$

9 points

(b) $\int_{2007}^{2007} \left(\sin(x)\cos(\cos(x)) + x^{13/18}\right)\,dx$

$= 0$

2 points

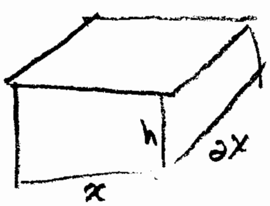
(c) $\int_{13}^{27} f(x)\,dx$, if $\int_1^{13} f(x)\,dx = -38$ and $\int_{27}^{1} f(x)\,dx = -96$.

$\int_{13}^{27} f(x)\,dx = \int_1^{27} f(x)\,dx - \int_1^{13} f\,dx = -\int_{27}^{1} f\,dx - \int_1^{13} f\,dx$

$= -(-96) - (-38) = 134$

6 points

8. Suppose that we wish to construct a rectangular box with *no lid.* The base of the box has sides in the ratio 2:1. The volume of the box is 36 in^3. Find the *dimensions* that minimize the material to make it.



$V = 2x^2h = 36 \Rightarrow h = \frac{18}{x^2}$

$A = 2x^2 + 2(xh) + 2(2xh)$

$A = 2x^2 + 6xh$

$A(x) = 2x^2 + \frac{108}{x}$

$A' = 4x - \frac{108}{x^2} = \frac{4x^3 - 108}{x^2} = 0$

$x = \sqrt[3]{\frac{108}{4}} = 3$

$h = \frac{18}{3^2} = 2$

$x = 3,\ 2x = 6,\ h = 2$

10 points

Page 5 Total (27 pts)