Calc I - 3450:221
EXAM #3 F05

NAME Key
ROW

100 Points

Show all your work.

1. (6 pts) Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of

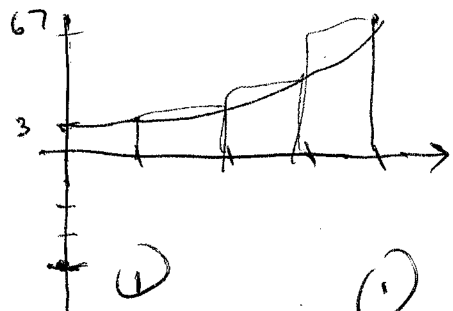
$$y = \int_{\cos x}^{x^2} (t^2 + 4)\, dt$$

$$\frac{dy}{dx} = (x^4 + 4) \cdot 2x - (\cos^2 x + 4) \cdot (-\sin x)$$

1 1 1 1 2

6 Points

2. (7 pts) Estimate the area under the graph of $f(x) = 4x^2 + 3$ from $x = 0$ to $x = 4$ using 4 equal subinterval rectangles and right endpoints.



1 1 1 1

$$A \approx (4+3)\cdot 1 + (4\cdot 4+3)\cdot 1 + (4\cdot 9+3)\cdot 1 + (4\cdot 16+3)\cdot 1$$

$$= 7 + 19 + 39 + 67 = 132 \quad 3$$

7 Points

3. (9 pts) Find f(x) if $f''(x) = -3\cos(x) + 2\sin(x) + 2$, subject to $f(0) = 8$, and $f'(0) = -8$.

$f'(x) = -3\sin x - 2\cos x + 2x + a$ 3

$f'(0) = -2 + a = -8 \Rightarrow a = -6$ 1

$f(x) = 3\cos x - 2\sin x + x^2 - 6x + b$ 4

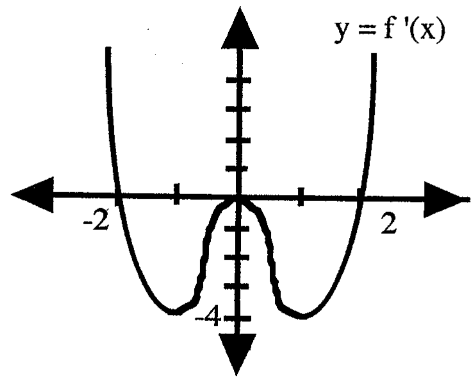
$f(0) = 3 + b = 8 \Rightarrow b = 5$ 1

so $f(x) = 3\cos x - 2\sin x + x^2 - 6x + 5$

9 Points

22 Points

4. The graph of the **derivative** f ' of a continuous function f is shown. Answer the following questions:



a. (3 pts) On what intervals is f increasing or decreasing?

inc $(-\infty, -2) \cup (2, \infty)$

dec $(-2, 0) \cup (0, 2)$

b. (1 pt) At what values of x does f have a local maximum?

$x = -2$

c. (1 pt) At what values of x does f have a local minimum?

$x = 2$

d. (4 pts) On what intervals is f concave upward or downward?

up on $(-1, 0) \cup (1, \infty)$

down $(-\infty, -1) \cup (0, 1)$

e. (3 pts) What are the x-coordinates of the points of inflection?

$x = -1, 0, 1$

12 Points

5. Evaluate the following integrals:

a. (2 pts) $\int_6^6 (4x^2 - 5)^3 \, dx$

or

equal limits 0

b. (5 pts) $\int \left(\frac{x^3 - 4x + 7}{\sqrt{x}} \right) dx$

$\int \left[x^{5/2} - 4x^{1/2} + 7x^{-1/2} \right) dx$ 1

$= \frac{2}{7} x^{7/2} - \frac{4 \cdot 2}{3} x^{3/2} + 7 \cdot 2 x^{1/2} + C$

4

c. (4 pts) $\int_2^3 \left(\frac{x^2 - 1}{x - 1} \right) dx$

$\int_2^3 (x+1) \, dx$

$= \frac{x^2}{2} + x \Big|_2^3$ 2

$= \left(\frac{9}{2} + 3 \right) - \left(\frac{4}{2} + 2 \right)$

$= \frac{9}{2} - 1 = 7/2$

2

d. (4 pts) $\int_0^{\pi/4} \sec^2 x \, dx$

$\tan x \Big|_0^{\pi/4}$ 2

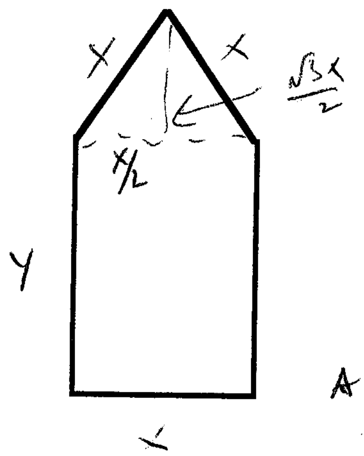
$= \tan \pi/4 - \tan 0$

$= 1$ 2

15 Points

27 Points

6. (10 pts) A window has the shape of a rectangle surmounted by an equilateral triangle as shown in the figure. If the perimeter of the window is 12 ft, then find the dimensions of the window leading to the maximum area of the window.



$3x + 2y = 12 \Rightarrow y = 6 - \frac{3}{2}x$ 3

$A = xy + \frac{1}{2}x \cdot \frac{\sqrt{3}x}{2}$

$= x(6 - \frac{3}{2}x) + \frac{\sqrt{3}}{4}x^2$ 4

$= 6x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$ 3

$A' = 6 - 3x + \frac{\sqrt{3}}{2}x = 0 \Rightarrow x = \frac{6}{3 - \sqrt{3}/2} = 2.812$

$A'' = -3 + \sqrt{3}/2 < 0$ so max

$y = 6 - \frac{3}{2} \cdot \left(\frac{6}{3 - \sqrt{3}/2}\right)$

10 Points

7. Given $f'(x) = \frac{(x-1)(x-5)}{(x-3)^2}$, $f''(x) = \frac{8}{(x-3)^3}$, $\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = \infty$

answer the following questions: 1 3 5

a. (6 pts) On what intervals is f increasing and decreasing?

x	x-1	x-5	f'	concl
$(-\infty, 1)$	$-$	$-$	$+$	inc
$(1, 3)$	$+$	$-$	$-$	dec
$(3, 5)$	$+$	$-$	$-$	dec
$(5, \infty)$	$+$	$+$	$+$	inc

max

min

b. (4 pts) What are the x-coordinates of any local maxima and local minima?

$x = 1$ local max 2 $x = 5$ local min 2

c. (4 pts) On what intervals is f concave upward or downward?

x	x-3	f''	concl
$(-\infty, 3)$	$-$	$-$	down 2
$(3, \infty)$	$+$	$+$	up 2

d. (2 pts) What are the x-coordinates of any inflection points?

no inflection points

16 Points

26 Points

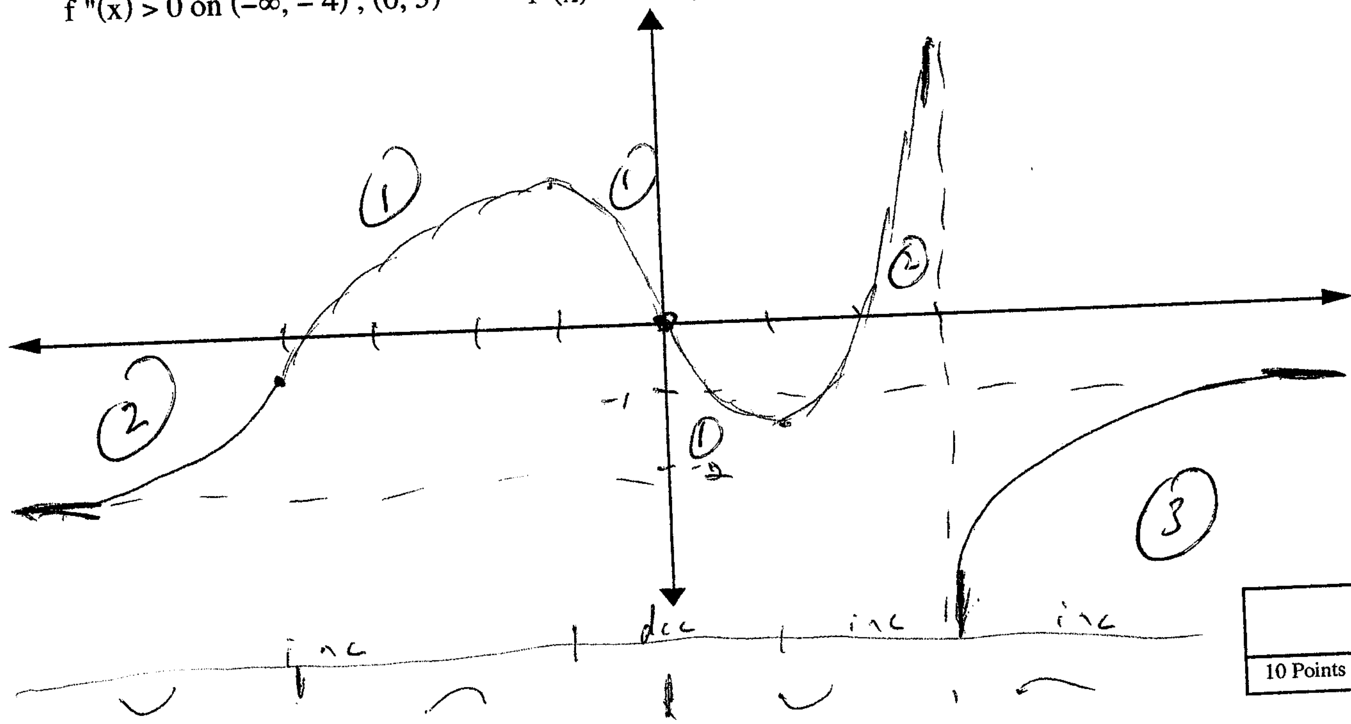
8. (10 pts) A function f satisfies the following properties. Sketch the graph of f.

$f(0) = 0$; $\lim_{x\to\infty} f(x) = -1$, $\lim_{x\to-\infty} f(x) = -2$; $\lim_{x\to 3^-} f(x) = \infty$, $\lim_{x\to 3^+} f(x) = -\infty$;

$f'(x) > 0$ on $(-\infty, -1)$, $(1, 3)$, $(3, \infty)$

$f'(x) < 0$ on $(-1, 1)$

$f''(x) > 0$ on $(-\infty, -4)$, $(0, 3)$ $\quad$ $f''(x) < 0$ on $(-4, 0)$, $(3, \infty)$



10 Points

9. Evaluate the following limits:

a. (8 pts) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n} + \frac{3i^2}{n^2}\right)\left(\frac{4}{n}\right)$

$\lim_{n\to\infty} \frac{4}{n} \cdot \frac{2}{n} \sum_{i=1} i + \frac{4}{n} \cdot \frac{3}{n^2} \sum_{i=1} i^2$

$= \lim_{n\to\infty} \left[\frac{8}{n^2} \cdot \frac{n(n+1)}{2} + \frac{12}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right]$ (2) (2)

$= 4 + \frac{24}{6}$ (2) (2)

$= 4 + 4 = 8$

b. (7 pts) $\lim_{x\to-\infty} \left(4x + \sqrt{16x^2 + 4x - 3}\right)$

$\lim_{n\to-\infty} \frac{(4x + \sqrt{\ })(4x - \sqrt{\ })}{4x - \sqrt{16x^2 + 4x - 3}}$ (2)

$= \lim_{n\to-\infty} \frac{16x^2 - (16x^2 + 4x - 3)}{4x - \sqrt{x^2\left(16 + \frac{4}{x} - \frac{3}{x^2}\right)}}$

$= \lim_{n\to-\infty} \frac{-4x + 3}{4x - |x|\sqrt{16 + \frac{4}{x} - 3/x^2}}$ (2)

$= \lim_{n\to-\infty} \frac{-4x/x + 3}{4x/x + x\sqrt{16 + \frac{4}{x} - 3/x^2}}$

(3) $= \frac{-4}{4+4} = -1/2$

15 Points

25 Points