CALCULUS I 3430.221 CLASS TIME ______________

TEST #3 SPRING 04

NAME ____

100 Points

Show all your work.

1. (4 pts) Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of

$$y = \int_{\cot(x)}^{2} (3x + 1)dx \;=\; -\int_{2}^{\cot(x)} (3x+1)\,dx$$

$$y' = -\left[3\cot(x) + 1\right](-\csc^2 x)$$

$$= (3\cot(x)+1)(\csc^2 x)$$

4 Points

2. (7 pts) Estimate the area under the curve of $f(x) = 2x^2 + 1$ from $x = -2$ to $x = 6$ using 4 rectangles and right endpoints. $\Delta x = \dfrac{6-(-2)}{4} = 2$

$$A \approx 2\left[f(0) + f(2) + f(4) + f(6)\right]$$

$$= 2\left[1 + 9 + 33 + 73\right]$$

$$= 232$$

7 Points

3. (6 pts) Find $f(x)$ if $f'(x) = \cos(x) + \csc^2(x)$, $0 < x < \pi$, and $f\left(\frac{\pi}{2}\right) = 5$

$$f(x) = \sin(x) - \cot(x) + C$$

$$5 = 1 - 0 + C$$

$$C = 4$$

$$f(x) = \sin(x) - \cot(x) + 4$$

6 Points

4. (6 pts) If a ball is thrown upward with a velocity of 60 ft/sec from the top of a 44 foot tall building, find the height of the ball at time t.

$$s''(t) = -32 \text{ ft/s}^2$$

$$s'(t) = -32t + C \qquad s'(0) = 60$$

$$C = 60$$

$$s'(t) = -32t + 60$$

$$s(t) = -16t^2 + 60t + D \qquad s(0) = 44$$

$$D = 44$$

$$s(t) = -16t^2 + 60t + 44$$

6 Points

5. (9 pts) Sketch the graph of the function f that satisfies the following:

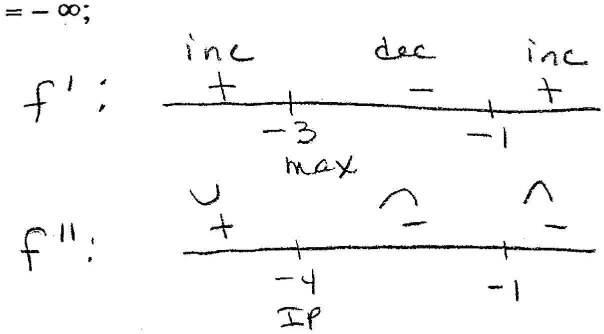
$f(0) = 0$; $\lim_{x\to\infty} f(x) = 2$; $\lim_{x\to-\infty} f(x) = 2$; $\lim_{x\to-1} f(x) = -\infty$;

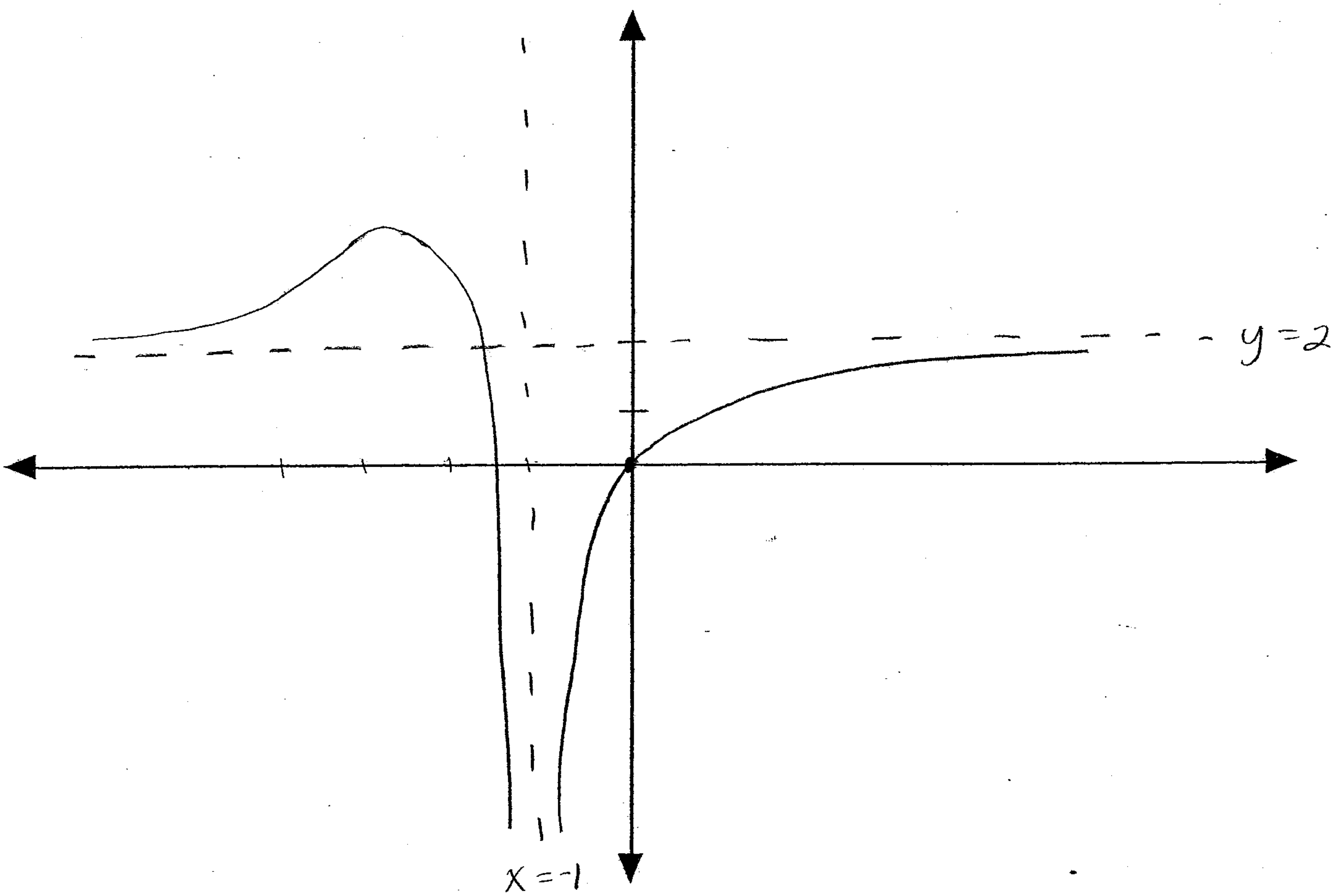
$f'(x) > 0$ on $(-\infty, -3) \cup (-1, \infty)$

$f'(x) < 0$ on $(-3, -1)$

$f''(x) > 0$ on $(-\infty, -4)$

$f''(x) < 0$ on $(-4, -1) \cup (-1, \infty)$





9 Points

6. Given $f'(x) = \dfrac{9 - x^2}{(x^2 + 9)^2}$ and $f''(x) = \dfrac{2x(x^2 - 27)}{(x^2 + 9)^3}$ answer the following questions:

a. (6 pts) Find the intervals where f(x) is increasing and decreasing.

$$9 - x^2 = 0$$
$$x = \pm 3$$

f': $-$ | -3 | $+$ | 3 | $-$

Inc: $(-3, 3)$

Dec: $(-\infty, -3) \cup (3, \infty)$

b. (3 pts) At what values of x do the local maximum and local minimum occur? (Label each)

Local Max $x = 3$

Local Min $x = -3$

c. (6 pts) Find the intervals of concavity.

$x = 0$ $\quad$ $x^2 = 27$

$x = \pm\sqrt{27}$

$x = \pm 3\sqrt{3}$

f'': $-$ | $-3\sqrt{3}$ | $+$ | 0 | $-$ | $3\sqrt{3}$ | $+$

concave ↑

$(-3\sqrt{3}, 0) \cup (3\sqrt{3}, \infty)$

concave ↓

$(-\infty, -3\sqrt{3}) \cup (0, 3\sqrt{3})$

d. (3 pts) At what values of x do the inflection points occur?

$x = 0, \pm 3\sqrt{3}$

18 Points

7. **Evaluate the following limits:**

a. (3 pts) $\lim_{x\to\infty}\left(\frac{5x^3+2x-1}{x-7x^3}\right)$

$$= \lim_{x\to\infty}\left(\frac{5x^3}{-7x^3}\right)$$

$$= -\frac{5}{7}$$

b. (10 pts) $\lim_{x\to-\infty}\left(\sqrt{64x^2-3x}+8x\right)\cdot\frac{\sqrt{64x^2-3x}-8x}{\sqrt{64x^2-3x}-8x}$

$$= \lim_{x\to-\infty}\left[\frac{64x^2-3x-64x^2}{\sqrt{64x^2-3x}-8x}\right]$$

$$= \lim_{x\to-\infty}\left[\frac{-3x}{\sqrt{x^2(64-\frac{3}{x})}-8x}\right]$$

$$= \lim_{x\to-\infty}\left[\frac{-3x}{|x|\sqrt{64-3/x}-8x}\right]$$

$$= \lim_{x\to-\infty}\left[\frac{-3}{\frac{|x|}{x}\sqrt{64-3/x}-8}\right]$$

$$= \frac{-3}{(-1)(8)-8} = \frac{-3}{-16} = \frac{3}{16}$$

c. (10 pts) $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{4i}{n}\right)^3+1\right]\left(\frac{4}{n}\right)$

$$= \lim_{n\to\infty}\left(\frac{4}{n}\right)\sum_{i=1}^{n}\left(\frac{64i^3}{n^3}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{4}{n}\right)\left[\sum_{i=1}^{n}\frac{64i^3}{n^3}+\sum_{i=1}^{n}1\right]$$

$$= \lim_{n\to\infty}\left(\frac{4}{n}\right)\left[\frac{64}{n^3}\sum_{i=1}^{n}i^3+n\right]$$

$$= \lim_{n\to\infty}\left(\frac{4}{n}\right)\left[\frac{64}{n^3}\left(\frac{n(n+1)}{2}\right)^2+n\right]$$

$$= \lim_{n\to\infty}\left[\frac{4\cdot 64\cdot n^4}{4n^4}+4\right]$$

$$= 64+4$$

$$= 68$$

23 Points

8. Evaluate the following integrals:

a. (9 pts) $\int_1^4 \left(\frac{x^3 + 3\sqrt{x}}{x^3} \right) dx$

$$= \int_1^4 \left(1 + 3x^{-5/2}\right) dx$$

$$= \left[x + 3\left(-\tfrac{2}{3}\right)x^{-3/2}\right]_1^4$$

$$= \left[x - 2x^{-3/2}\right]_1^4$$

$$= 4 - 2\left(\tfrac{1}{8}\right) - (1 - 2)$$

$$= 4 - \tfrac{1}{4} + 1$$

$$= 19/4$$

b. (2 pts) $\int_{\pi/2}^{\pi/2} \sin(x)\, dx$

$$= 0$$

11 Points

9. (6 pts) Find $\int_1^5 f(x)\, dx$ if $\int_1^2 f(x)\, dx = 3$ and $\int_5^2 f(x)\, dx = -8$

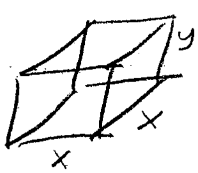
$$\int_1^5 f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^5 f(x)\,dx$$

$$= 3 - \int_5^2 f(x)\,dx$$

$$= 3 - (-8) = 11$$

6 Points

10. (10 pts) A rectangular box is to have a square base and a volume of 20 cubic feet. If the material for the base costs 30 cents per square foot, the material for the sides costs 10 cents per square foot, and the material for the top costs 20 cents per square foot, determine the dimensions of the box that can be constructed at minimum cost.



$$x^2 y = 20$$

$$y = \frac{20}{x^2}$$

$$C = 30x^2 + 10(4)xy + 20x^2$$

$$= 50x^2 + 40x\left(\frac{20}{x^2}\right)$$

$$= 50x^2 + 800x^{-1}$$

$$C' = 100x - 800x^{-2}$$

$$100x = \frac{800}{x^2}$$

$$100x^3 = 800$$

$$x^3 = 8$$

$$x = 2 \text{ ft.}$$

$$y = 5 \text{ ft}$$

$-$ $+$

2

abs. min.

10 Points