Avg 79
min 54
med 80
max 96
n = 29

Honors Calculus I : Fall 2007
Exam 2

Test Total

Name KEY

INSTRUCTIONS : Show all of your work.

1. Find the *critical numbers* for the function $f(t) = t^{1/3}(10-t)^{1/5}$.

$$f' = \frac{1}{3}t^{-2/3}(10-t)^{1/5} + t^{1/3}\cdot\frac{1}{5}(10-t)^{-4/5}\cdot -1 \quad (4)$$

$$= t^{-2/3}(10-t)^{-4/5}\left(\frac{1}{3}(10-t) - t\cdot\frac{1}{5}\right)$$

$$= \frac{\frac{10}{3} - \frac{8}{15}t}{t^{2/3}(10-t)^{4/5}} \quad (2)$$

$f' = 0$: $t = \frac{25}{4}$ (2) or 6.25

f' dne: $t = 0, 10$ (2)

10 points

2. Find and *simplify* y' for:

(a) $y = \sqrt{\sec(\cos(x^3))}$ $= (\sec(\cos(x^3)))^{1/2}$

$$y' = \frac{1}{2\sqrt{\sec(\cos(x^3))}} \cdot \sec(\cos(x^3))\cdot\tan(\cos(x^3))\cdot -\sin(x^3)\cdot 3x^2$$

(1) (1) (1) (1)

5 points

$$= -\frac{3}{2}x^2\sqrt{\sec(\cos x^3)}\cdot\tan(\cos x^3)\cdot\sin(x^3)$$

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(b) $y = \left(\dfrac{x^2-2}{x^2+x-1}\right)^4 = (x^2-2)^4(x^2+x-1)^{-4}$

$y' = 4(x^2-2)^3 \cdot 2x\,(x^2+x-1)^{-4} + (x^2-2)^4 \cdot -4\,(x^2+x-1)^{-5} \cdot (2x+1)$ (4)

$= (x^2-2)^3(x^2+x-1)^{-5}\left(4(2x)(x^2+x-1) + (x^2-2)(-4)(2x+1)\right)$

$= \dfrac{4(x-2)^3(x^2+2x+2)}{(x^2+x-1)^5}$ (1)

5 points

(c) $y = \cot\left(\dfrac{1}{\sqrt{x}}\right)$

$y' = -\csc\left(\frac{1}{\sqrt{x}}\right)\cot\left(\frac{1}{\sqrt{x}}\right) \cdot -\frac{1}{2}x^{-3/2} = \dfrac{1}{2\sqrt{x^3}}\csc\frac{1}{\sqrt{x}}\cot\frac{1}{\sqrt{x}}$

(4) (1)

5 points

3. Shari gets on the turnpike at exactly 1:37pm, as stamped on her ticket. She arrives at her exit, 18.6 miles away at exactly 1:53pm. The speed limit is 65 mph. Can the toll worker legitimately issue a speeding ticket for doing 70mph? Explain your answer carefully.

(4) $\dfrac{S(1{:}53) - S(1{:}37)}{16\text{ min} \times \frac{1\text{ hr}}{60\text{ min}}} = \dfrac{18.6\text{ mi}}{16\text{ min}} \times \dfrac{60\text{ min}}{\text{hr}} = 69.75\text{ mph.}$ (2)

10 points

Given her position is continuous & differentiable
(4) The mean Value Theorem Says at Some place
She was going 69.75 mph, The toll worker
Can not argue she was going 70 mph

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4. Consider the *implicit equation* $\tan(x - y^2) + 128 = x^3y$.

(a) Find and simplify $\dfrac{dy}{dx}$.

$$\sec^2(x-y^2)(1-2yy') + 0 = 3x^2y + x^3y' \quad ①①①①$$

$$(-2y\sec^2(x-y^2) - x^3)y' = 3x^2y - \sec^2(x-y^2) \quad 2$$

$$y' = \frac{3x^2y - \sec^2(x-y^2)}{-2y\sec^2(x-y^2) - x^3} \quad ④$$

10 points

(b) Find the equations of the *tangent line* and *normal line* at the point (4,2).

$$T: \ y-2 = -\frac{95}{68}(x-4) \quad ③$$

$$N: \ y-2 = \frac{68}{95}(x-4) \quad ③$$

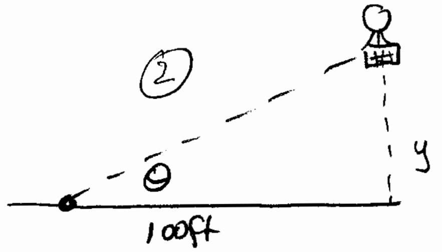
$$y'|_{(4,2)} = \frac{3(16)(2) - \sec^2(4-4)}{-2(2)\sec^2(4-4) - 4^3} = \frac{95}{-68} \quad ④$$

$$= -\frac{95}{68}$$

10 points

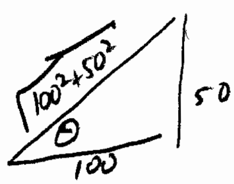
5. A man is lying on the ground 100 feet from a hot-air balloon launch. The balloon is rising at 10 feet per second. Determine the *rate of change* of the angle of view of the man from the horizontal to the balloon 5 seconds after launch.

②



$$① \ \frac{dy}{dt} = 10 \ \frac{ft}{sec}$$

$$① \ y = 10 \ ft/sec \cdot 5 \ sec = 50 \ ft$$



$$\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj} = \frac{\sqrt{100^2+50^2}}{100} \quad ②$$

$$③ \ \tan\theta = \frac{y}{100}$$

$$③ \ \sec^2\theta \frac{d\theta}{dt} = \frac{dy/dt}{100}$$

15 points

$$① \ \frac{d\theta}{dt} = \frac{10/100}{\frac{(\sqrt{100^2+50^2})^2}{100^2}} = \frac{10(100)}{100^2+50^2} = 0.08 \ \frac{rad}{sec} \quad ①$$

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6. Consider the function $g(\theta) = \cos\theta$.

(a) Construct the *linearization* of g at the point $\theta = \frac{\pi}{3}$.

$$L(x) = \overset{2}{\cos\frac{\pi}{3}} + -\overset{4}{\sin\left(\frac{\pi}{3}\right)}\overset{2}{\left(x - \frac{\pi}{3}\right)}$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) \quad 2$$

10 points

(b) Use the linearization to estimate the value of $\cos 59^o$.

$$L(59^o) = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(59^\circ \times \frac{\pi}{180^\circ} - \frac{\pi}{3}\right) \quad 3$$

$$= 0.51511 \quad 2$$

5 points

7. Determine the *exact* maximum and minimum of $f(x) = \dfrac{\cos x}{2 - \sin x}$ on the interval $[0, \pi]$.

$$f' = \frac{-\sin x(2 - \sin x) - \cos x(-\cos x)}{(2 - \sin x)^2} = \frac{-2\sin x + 1}{(2 - \sin x)^2} \quad 6$$

critical:

$f' = 0 \quad \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \in [0, \pi]$ 5

f' dne - none in domain of f.

$f(0) = \frac{1}{2} = .5$

$f\left(\frac{\pi}{6}\right) = \dfrac{\sqrt{3}/2}{2 - 1/2} = \sqrt{3}/3 = .577$ abs max 2

$f\left(\frac{5\pi}{6}\right) = \dfrac{-\sqrt{3}/2}{2 - 1/2} = -\sqrt{3}/3 = -.577$ abs min 2

$f(\pi) = -\frac{1}{2} = -.5$

15 points

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