Calc I - 3450:221
EXAM #2 F05

NAME Key
ROW

100 Points

Show **ALL** your work.

1. Find $\frac{dy}{dx}$ if $y = \sec\sqrt{\tan^2(3x-1) - \sin(x^2+2x-1)}$. Do not simplify your answer.

$$\frac{dy}{dx} = \sec\sqrt{\tan^2(3x-1) - \sin(x^2+2x-1)}\,\tan\sqrt{\tan^2(3x-1) - \sin(x^2+2x-1)}$$
$$\cdot \frac{1}{2}\left[\tan^2(3x-1) - \sin(x^2+2x-1)\right]^{-1/2} \cdot$$
$$\left[2\tan(3x-1)\cdot\sec^2(3x-1)\cdot 3 - \cos(x^2+2x+1)\cdot(2x+2)\right]$$

10 Points

2. Find $\frac{dy}{dx}$ if $y = \csc(x^2+9x-1)\cos^4(4x^{-1}+2)$. Do not simplify your answer.

$$\frac{dy}{dx} = \csc(x^2+9x-1)\cdot 4\cos^3(4x^{-1}+2)\cdot\left[-\sin(4x^{-1}+2)\cdot(-4x^{-2})\right]$$
$$+ \cos^4(4x^{-1}+2)\cdot\left[-\csc(x^2+9x+1)\cot(x^2+9x+1)\cdot(2x+9)\right]$$

10 Points

3. Find and simplify an expression for y'' given $y = x\sin(x^2)$.

$$y = x\sin(x^2)$$
$$\frac{dy}{dx} = x\cdot\cos(x^2)\cdot 2x + \sin(x^2)$$
$$= 2x^2\cos(x^2) + \sin(x^2)$$

10 Points

$$\frac{d^2y}{dx^2} = 2x^2(-\sin(x^2))\cdot 2x + 4x\cos(x^2) + \cos(x^2)\cdot 2x$$
$$= -4x^3\sin(x^2) + 6x\cos(x^2)$$

30 Points

4. Evaluate $\lim\limits_{x \to 0} \dfrac{\sin^2(3x)}{x \tan(5x)}$.

$$\lim_{x \to 0} 3\,\frac{\sin(3x)}{3x}\cdot\frac{\sin(3x)\,3x\cos(5x)\,5x}{3x\,\sin(5x)\,5x} = \frac{9}{5}$$

1 1 1

8 Points

5. Use differentials or the equivalent linearization to estimate the value of $\sin(59.8^{o})$.

$\sin(x) = \sin(a) + \cos(a)\,(x-a)$ (1)

Take $a = \pi/3$

$$\sin\left(59.8 \cdot \frac{\pi}{180}\right) = \sin(\pi/3) + \cos(\pi/3)\left(\frac{59.8\,\pi}{180} - \frac{60\pi}{180}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\left(-\frac{0.2\pi}{180}\right) = 0.86428$$

(2) (3) (3)

9 Points

6. The position of a particle is given by $s(t) = 2t^3 - 15t^2 + 36t + 2$, where s is measured in meters and t in seconds. Find the velocity when the acceleration is $-6\ m/s^2$.

$v(t) = 6t^2 - 30t + 36$ (3)

$a(t) = v'(t) = 12t - 30 = -6$ (3)

$12t = 24$

$t = 2s$ (2)

So $v(2) = 6 \cdot 4 - 60 + 36 = 0$ (2)

10 Points

27 Points

7. Find the absolute maximum and minimum of $y = \dfrac{x}{x^2 + 1}$ on the domain [–2, 2].

2

$\frac{dy}{dx} = \frac{(x^2+1)\cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$

4

critical numbers

10 Points

$y(1) = \frac{1}{1+1} = 1/2$ max

$y(-1) = \frac{-1}{1+1} = -1/2$ min

4

$y(-2) = \frac{-2}{4+1} = -2/5$

$y(2) = \frac{2}{4+1} = 2/5$

8. Find a general formula for $f^{(n)}(x)$ where $f(x) = \dfrac{1}{(3x + 4)^2}$.

$f(x) = (3x+4)^{-2}$

$f'(x) = -2\,(3x+4)^{-3} \cdot 3$

2

$f''(x) = (-2)(-3)\,(3x+4)^{-4} \cdot 3 \cdot 3$

$f'''(x) = (-2)(-3)(-4)\,(3x+4)^{-5} \cdot 3 \cdot 3 \cdot 3$

$\vdots$

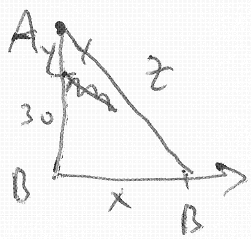
$f^{(n)}(x) = (-1)^n\,(n+1)!\,(3x+4)^{-(n+2)} \cdot 3^n$

2 2 2 2

10 Points

20 Points

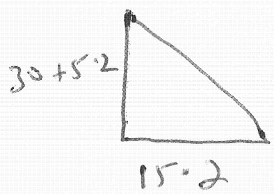
9. At noon ship A is 30 miles north of Ship B. Ship A is sailing north at a speed of 5 miles/hour. Ship B is sailing east at a speed of 15 miles/hour. How fast is the distance between the ships changing at 2:00 p.m.?

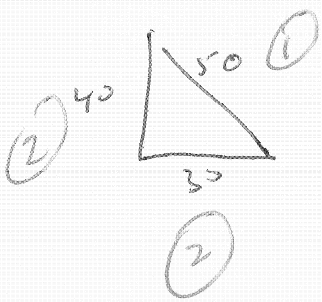


$$\frac{dy}{dt} = 5\,\frac{m}{hr}$$

$$\frac{dx}{dt} = \frac{15\,m}{hr}$$

13 Points





$$z^2 = x^2 + (30+y)^2 \quad 2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2(30+y)\frac{dy}{dt} \quad 3$$

$$\cancel{2}\cdot 50\,\frac{dz}{dt} = \cancel{2}\cdot 30\cdot 15 + \cancel{2}(30+10)\cdot 5 \quad 2$$

$$\frac{dz}{dt} = \frac{450+200}{50} = \frac{65\cancel{0}}{5\cancel{0}} = 13\,\frac{m}{hr}$$

1

10. Find the equation of the line normal to $4x^3 - 3xy^2 + y^3 = 49$ at the point $(-2, 3)$.

$$12x^2 - 3[x2yy' + y^2] + 3y^2y' = 0$$

$$12\cdot 4 - 3[-4\cdot 3y' + 9] + 27y' = 0$$

10 Points

$$48 + 36y' - 27 + 27y' = 0$$

$$63y' = 27 - 48 = -21$$

$$y' = -\frac{21}{63} = -1/3 \quad 2$$

So slope normal $= 3$

equation normal $\frac{y-3}{x+2} = 3$ 2

23 Points