CALCULUS I - 3430.221
TEST #2 SPRING 04

CLASS TIME ______________

NAME ____

100 Points

Show all your work.

1. Find the derivative of the following functions. **DO NOT** simplify your answer.

a. (7 pts) $f(x) = (2x^3 + 1)^4 \tan(3x^2 - \sqrt{x})$

$$f'(x) = (2x^3+1)^4 \sec^2(3x^2 - \sqrt{x})(6x - \tfrac{1}{2}x^{-1/2}) + \tan(3x^2 - \sqrt{x})(4)(2x^3+1)^3(6x^2)$$

b. (7 pts) $f(x) = \sec^2(\sin(2x-1)^5)$

$$f'(x) = 2\sec(\sin(2x-1)^5)\sec(\sin(2x-1)^5)\tan(\sin(2x-1)^5) \cos(2x-1)^5 (5)(2x-1)^4(2)$$

14 Points

2. (11 pts) Find y " if $x^3 - y^3 = 5$

$$3x^2 - 3y^2 y' = 0$$

$$x^2 - y^2 y' = 0$$

$$x^2 = y^2 y'$$

$$y' = \frac{x^2}{y^2}$$

$$y'' = \frac{y^2(2x) - x^2(2yy')}{y^4} = \frac{2xy^2 - 2x^2y\left(\frac{x^2}{y^2}\right)}{y^4}$$

$$= \frac{2xy^2 - \frac{2x^4}{y}}{y^4} \cdot \frac{y}{y} = \frac{2xy^3 - 2x^4}{y^5} = \frac{2x(y^3 - x^3)}{y^5}$$

$$= \frac{2x(-5)}{y^5} = \frac{-10x}{y^5}$$

11 Points

3. (7 pts) Evaluate $\lim_{x\to 0}\left[\frac{\sin^2 7x}{\sin^2 5x}\right]$

$$= \left[\lim_{x\to 0}\left(\frac{\sin 7x}{\sin 5x}\right)\right]^2 = \left[\lim_{x\to 0} \frac{\frac{\sin 7x}{7x}\cdot\frac{7}{1}}{\frac{\sin 5x}{5x}\cdot\frac{5}{1}}\right]^2 = \left(\frac{7}{5}\right)^2 = \frac{49}{25}$$

7 Points

4. (9 pts) Use differentials or the equivalent linearization to approximate tan(59°).

$dy = f'(x)\,dx \qquad f(x) = \tan x$

$\qquad\qquad\qquad f'(x) = \sec^2 x$

$dy = \sec^2 x \; dx \qquad x = 60° \qquad dx = (-1°)\left(\frac{\pi}{180°}\right) = \frac{-\pi}{180}$

$dy = \sec^2(60°)\left(-\frac{\pi}{180}\right) = 4\left(\frac{-\pi}{180}\right) = -\frac{\pi}{45}$

$\tan(59°) \approx \tan 60° + dy = \sqrt{3} - \frac{\pi}{45} \approx 1.6622$

9 Points

5. (8 pts) Find a formula for the nth derivative of $y = \dfrac{3}{(1+2x)^4}$ $= 3(1+2x)^{-4}$

$n=1 \qquad f'(x) = 3(-4)(1+2x)^{-5}(2)$

$n=2 \qquad f''(x) = 3(-4)(-5)(1+2x)^{-6}(2)(2)$

$n=3 \qquad f'''(x) = 3(-4)(-5)(-6)(1+2x)^{-7}(2)(2)(2)$

$\vdots$

8 Points

$f^{(n)}(x) = \cancel{3}(-1)^n \dfrac{(n+3)!}{\cancel{3}\cdot 2\cdot 1}(1+2x)^{-(n+4)}\,2^n$

$= (-2)^n \dfrac{(n+3)!}{2\cdot 1}(1+2x)^{-(n+4)}$

6. (10 pts) Show that the families of curves given by $\overset{①}{y = ax^3}$ and $\overset{②}{x^2 + 3y^2 = b}$ are orthogonal trajectories of each other.

① $y' = 3ax^2$

② $2x + 6yy' = 0$

$x + 3yy' = 0$

$3yy' = -x$

$y' = -\dfrac{x}{3y}$

10 Points

let (x_0, y_0) be pt. of intersection

$3ax_0^2 = \dfrac{3y_0}{x_0}$

$3ax_0^3 = 3y_0$

$ax_0^3 = y_0$ true by ①

therefore orthogonal trajectories

7. A particle moves according to a law of motion $s(t) = 2t^3 - 3t^2$, $t \geq 0$ where t is measured in seconds and s in feet.

a. (2 pts) Find the velocity at time t.

$$v(t) = 6t^2 - 6t$$

b. (2 pts) Find the acceleration at time t.

$$a(t) = 12t - 6$$

c. (4 pts) Find the time(s) when the particle is at rest. $v(t) = 0$

$$6t^2 - 6t = 0$$
$$6t(t-1) = 0$$
$$t = 0 \text{ sec.} \qquad t = 1 \text{ sec}$$

d. (3 pts) Find when the particle is moving in the positive direction.

$$v(t) > 0$$

$-$ $+$

0 $t = 1/2$ 1 $t = 2$

$(1, \infty)$ sec.

e. (4 pts) Find the acceleration when the velocity is 12 feet per second.

$$6t^2 - 6t = 12$$
$$6t^2 - 6t - 12 = 0$$
$$t^2 - t - 2 = 0$$
$$(t-2)(t+1) = 0$$

$t = 2$ sec. $\quad t = \cancel{-1 \text{ sec.}}$

$$a(2) = 24 - 6$$
$$= 18 \text{ ft/sec}^2$$

15 Points

8. (6 pts) Find the critical numbers of $f(x) = 2x - 3x^{2/3}$

$$f'(x) = 2 - 2x^{-1/3}$$
$$= 2 - \frac{2}{\sqrt[3]{x}}$$

$$2 - \frac{2}{\sqrt[3]{x}} = 0$$
$$2 = \frac{2}{\sqrt[3]{x}}$$
$$2\sqrt[3]{x} = 2$$
$$\sqrt[3]{x} = 1$$
$$x = 1$$

6 Points

critical numbers are $x = 0$, $x = 1$

9. (10 pts) Find the absolute maximum and absolute minimum values (label each) of

$$f(x) = 3x^4 - 4x^3 \quad \text{on } [-1, 2]$$

$$f'(x) = 12x^3 - 12x^2$$

$$12x^2(x-1) = 0$$

$$x = 0 \qquad x = 1$$

$f(0) = 0$

$f(1) = -1$ abs. min.

$f(-1) = 3+4 = 7$

$f(2) = 3(16) - 4(8) = 48-32 = 16$ abs max.

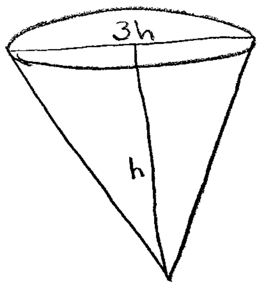
10 Points

10. (10 pts) Sand is falling off a conveyor and onto a conical pile at the rate of $10 ft^3/min$. The diameter of the base of the cone is 3 times the altitude. At what rate is the height of the pile changing when it is 15 feet high?

$\left(V = \frac{1}{3}\pi r^2 h\right)$

$$\frac{dV}{dt} = 10 \frac{ft^3}{min}$$

find $\frac{dh}{dt}$ when $h = 15$ ft.



$d = 3h$

$r = \frac{3h}{2}$

$$V = \frac{\pi}{3}\left(\frac{3h}{2}\right)^2 h$$

$$V = \frac{\pi}{3}\left(\frac{9h^2}{4}\right) h$$

$$V = \frac{3\pi}{4} h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{4} h^2 \frac{dh}{dt}$$

$$10 = \frac{9\pi}{4}(15)^2 \frac{dh}{dt}$$

$$10 = \left(\frac{9\pi}{4}\right)(225) \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{10(4)}{9\pi(225)} \approx 0.0063 \text{ ft/min.}$$

10 Points