Calculus I EXAM II Name: KEY

Fall 03 Oct. 10 Dr. Clemons

Directions: All work should be written in a neat and logical order with answers written in the allocated space. Should more space be needed, direct the grader to the back of the page.

1. [12 pts.]

a. If $f(2) = 2$, $g(2) = 3$, $f'(2) = 4$ and $g'(2) = 5$ then $\left[\frac{f}{g}\right]'(2) =$ $\frac{2}{9}$

$$\frac{f'(2)\,g(2) - f(2)g'(2)}{(g(2))^2} = \frac{4\cdot 3 - 2\cdot 5}{9} = \frac{2}{9}$$

b. If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial of degree n, then $\frac{d^n p(x)}{dx^n} =$ $a_n \cdot n!$

c. Does the function $f(x) = (3x-2)^{2/3}$ have a tangent line at $x = 2/3$? No. $f'(\frac{2}{3})$ dne.

d. $\lim_{x\to 0} \frac{\sqrt{3}x}{\sin\sqrt{2}x} =$ $\frac{\sqrt{3}}{\sqrt{2}}$.

2. [12 pts.] Use the limit definition of the derivative to compute $f'(x)$ for $f(x) = \cos(4x)$.

$$f' = \lim_{h\to 0} \frac{\cos(4(x+h)) - \cos 4x}{h} = \lim_{h\to 0} \frac{\cos 4x \cos 4h - \sin 4x \sin 4h - \cos 4x}{h}$$

$$= \lim_{h\to 0} \cos 4x\left(\frac{\cos 4h - 1}{h}\right) - \sin 4x \frac{\sin 4h}{h}$$

$$= 4\cos 4x \lim_{h\to 0}\left(\frac{\cos 4h - 1}{4h}\right) - 4\sin 4x \lim_{h\to 0} \frac{\sin 4h}{4h}$$

$$= 4\cdot\cos 4x \cdot 0 \quad - 4\cdot\sin 4x\cdot 1$$

$$= -4\sin 4x$$

3. [12 pts.] Find an equation for the tangent line to the graph of $x^2y^2 = (y+1)^2(2-y^2)$ at the point $(0, -2)$.

$$T: \; y+2 = y'\big|_{(0,-2)}(x-0)$$

$$2xy^2 + 2x^2y\frac{dy}{dx} = 2(y+1)\frac{dy}{dx}(2-y^2) + (y+1)^2(-2y)\frac{dy}{dx}$$

$$2x^2y - 2(y+1)(2-y^2) + 2y(y+1)^2\frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y - 2(y+1)(2-y^2) + 2y(y+1)^2}$$

Tangent line d.n.e. Since $(0,-2)$ is not on curve.

4. [12 pts.] Find the first and second derivative for the function $y = \dfrac{2x+3}{\sqrt{x^2+9}}$, [*Simplify your answers*].

$$y' = \frac{2\sqrt{x^2+9} - (2x+3)\cdot\frac{1}{2\sqrt{x^2+9}}\cdot 2x}{\left(\sqrt{x^2+9}\right)^2} = \frac{2(x^2+9) - 2x^2 - 3x}{(x^2+9)^{3/2}} = \frac{-3x+18}{(x^2+9)^{3/2}}$$

$$y'' = \frac{-3(x^2+9)^{3/2} - (-3x+18)\cdot\frac{3}{2}(x^2+9)^{1/2}\cdot 2x}{(x^2+9)^3}$$

$$= \frac{-3(x^2+9) - (-3x+18)\cdot 3x}{(x^2+9)^{5/2}}$$

$$= \frac{6x^2 - 54x - 27}{(x^2+9)^{5/2}}$$

5. [27 pts.] Compute the derivatives of the following functions [*Do not simplify*]:

(a) $f(x) = \cot^3(2x^3)$

$$f' = 3\cot^2(2x^3) \cdot -\csc^2(2x^3) \cdot 6x^2$$

(b) $g(x) = \sqrt{\dfrac{x^3}{\sec^5(x^3)}}$ $= x^{3/2}\,\mathrm{Sec}^{-5/2}(x^3) = x^{3/2}\,\mathrm{Co}^{5/2}(x^3)$

$$g' = \frac{3}{2}x^{1/2}\,\mathrm{Co}^{5/2}(x^3) + x^{3/2} \cdot \frac{5}{2} \cdot (\mathrm{Co}^{3/2}(x^3)) \cdot -\mathrm{S}(x^3) \cdot 3x^2$$

(c) $h(t) = (6t^2 + 5)^3(t^3 - 7)^{-4}$

$$h' = 3(\underline{6t^2+5})^2 \cdot 12t \cdot (t^3-7)^{-4} + (6t^2+5)^3 \cdot -4(t^3-7)^{-5} \cdot 3t^2$$

6. [10 pts.] Use a linear approximation to estimate $\sqrt[3]{0.98} + \sqrt[4]{0.98}$

$$f(x) = \sqrt[3]{x} + \sqrt[4]{x}$$

$$a = 1$$

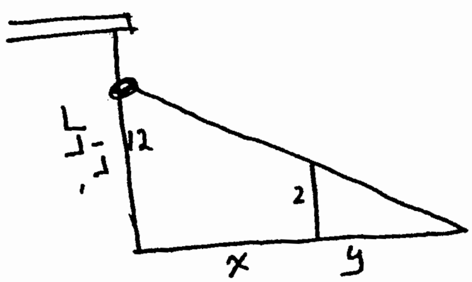
$$f' = \frac{1}{3x^{2/3}} + \frac{1}{4x^{3/4}}$$

$$L(x) = f(1) + f'(1)(x-1)$$

$$L(.98) \approx \left(\sqrt[3]{1} + \sqrt[4]{1}\right) + \left(\frac{1}{3 \cdot 1^{2/3} + 4 \cdot 1^{3/4}}\right)(0.98-1)$$

$$= 2 + \left(\tfrac{1}{3} + \tfrac{1}{4}\right)(-.02) = 1.9883$$

7. [15 pts.] A spoltight is located on a building 12 m high. If a man 2 m tall walks away from the spotlight at a speed of 1.6 m/s, find the rate of change of his shadow when he is 4 m from the building.



$$\frac{12}{2} = \frac{x+y}{y} \implies y = \tfrac{1}{5}x. \quad \text{Know } \frac{dx}{dt} = 1.6 \text{ when } x = 4$$

want $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{5}\frac{dx}{dt}\bigg|_{\substack{x=4 \\ \frac{dx}{dt}=1.6}} = \frac{1}{5}(1.6) = .32 \text{ m/s}.$$