$n = 30$
$\bar{x} = 90$
$\sigma = 9$
min = 57
med = 92.75
max = 9[illegible]

Test Total

Honors Calculus I : Fall 2007
Exam 1

Name KEY

INSTRUCTIONS : Show all of your work.

1. a) $\displaystyle\lim_{x\to -3}\left(\frac{2x^2+3}{x^3-1}\right)^2$

$= \left(\dfrac{2(-3)^2+3}{(-3)^3-1}\right)^2$

$= \left(\dfrac{21}{-28}\right)^2 = \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

b) $\displaystyle\lim_{h\to 0}\frac{h}{\sqrt{4+3h}-2} \times \frac{\sqrt{4+3h}+2}{\sqrt{4+3h}+2}$

$= \displaystyle\lim_{h\to 0}\frac{h(\sqrt{4+3h}+2)}{4+3h-4} = \lim_{h\to 0}\frac{\sqrt{4+3h}+2}{3}$

$= \dfrac{4}{3}$

c) $\displaystyle\lim_{x\to -3^-}\frac{|x+3|}{x+3}$

$= \displaystyle\lim_{x\to -3^-}\frac{-(x+3)}{x+3}$

$= \displaystyle\lim_{x\to -3^-} -1$

$= -1$

d) $\displaystyle\lim_{x\to 1}\frac{x^2+2x-3}{(x-1)^2}$

dne

20 points

2. Let $g(x) = \begin{cases} x^2-1 & \text{if } x < 2 \\ k & \text{if } 2 = x \\ 4x-5 & \text{if } 2 < x < 3 \\ \frac{8}{2x-3} & \text{if } 3 \le x \end{cases}$

(a) Find the value of k (if possible) that makes $g(x)$ continuous at $x = 2$.

$g(2) = k$ $\quad \displaystyle\lim_{x\to 2^-} g = \lim_{x\to 2^-} x^2-1 = 4-1 = 3$

$\displaystyle\lim_{x\to 2^+} g = \lim_{x\to 2^+} 4x-5 = 3$ $\quad$ so $k = 3$.

(b) Determine if $g(x)$ is continuous at $x = 3$, or determine the type of discontinuity. If not continuous, is $g(x)$ continuous from the right, left, or neither?

$g(3) = \dfrac{8}{2(3)-3} = \dfrac{8}{3}$

$\displaystyle\lim_{x\to 3^-} g = \lim_{x\to 3^-} 4x-5 = 7$

$\neq$

$\displaystyle\lim_{x\to 3^+} g = \lim_{x\to 3^+}\frac{8}{2x-3} = \frac{8}{3}$

Cont. from rt.

Discontinuous – jump in graph

10 points

Page 1 Total (30)

3. *Prove* that the equation $\sqrt{x} = 1 - \sin(x)$ has at least one solution in the interval $\left[0, \frac{\pi}{2}\right]$.

Consider $f(x) = \sqrt{x} - 1 + \sin x$, f is contin. on $[0, \infty)$

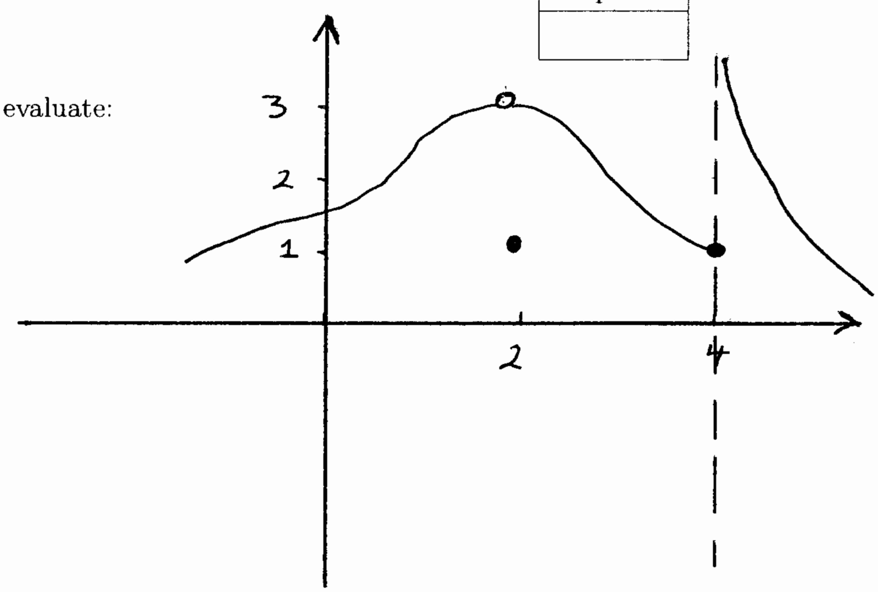
$f(0) = -1 < 0$

$f(\pi/2) = \sqrt{\pi/2} > 0$

By I.V.T. there exists a zero in $(0, \pi/2)$

10 points

4. Use the graph of $f(x)$ on the right to evaluate:



(a) $\lim_{x \to 2^-} f(x) =$ 3

(b) $\lim_{x \to 2^+} f(x) =$ 3

(c) $\lim_{x \to 2} f(x) =$ 3

(d) $\lim_{x \to 4^-} f(x) =$ 1

(e) $\lim_{x \to 4^+} f(x) =$ ∞ or dne

(f) $\lim_{x \to 4} f(x) =$ dne

12 points

5. Use the definition to find $h'(x)$, where $h(x) = \frac{1}{x^2}$.

$$h'(x) = \lim_{h \to 0} \frac{h(x+h) - h(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h\, x^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{h\, x^2 (x+h)^2} = \lim_{h \to 0} \frac{-2x - h}{x^2 (x+h)^2} = \frac{-2x}{x^2 (x)^2}$$

$$= \frac{-2}{x^3} .$$

10 points

Page 2 Total (32)

6. Find the *equation* of the tangent line to $j(x) = 2x^4 - 3x^2 + 9$ when $x = 1$.

$j(1) = 2 - 3 + 9 = 8$

T: $y - 8 = j'(1)(x - 1)$

$j' = 8x^3 - 6x$

$j'(1) = 8 - 6 = 2$

T: $y - 8 = 2(x - 1)$

$y = 2x + 6$

8 points

7. For $t \geq 0$ seconds, a particle has x-coordinate $s(t) = -2t^3 + 3t^2 + 12t + 6$ meters. Find the time(s) when the particle is moving to the *left*.

$s' = -6t^2 + 6t + 12 = -6(t^2 - t - 2) = -6(t-2)(t+1)$

s' : 0 at -1, 0 at 2, $- - -$ after 2; marks -1, 0, 2

$t > 2$

or

$(2, \infty)$

10 points

Page 3 Total (18)

8. Suppose that $f(x)$ and $g(x)$ are differentiable functions, and $f(2) = 3$, $g(2) = -1$, $f'(2) = 4$, $g'(2) = 1$.

Evaluate

a) $(fg)'(2)$

$f'(2)g(2) + f(2)g'(2)$

$= 4(-1) + 3(1)$

$= -1$

b) $\left(\dfrac{2f}{f+g}\right)'(2)$.

$$= \frac{2f'(2)(f(2)+g(2)) - 2f(2)(f'(2)+g'(2))}{(f(2)+g(2))^2}$$

$$= \frac{2(4)(3-1) - 2(3)(4+1)}{(3-1)^2} = \frac{16-30}{4}$$

$$= -\frac{14}{4} = -\frac{7}{2}$$

10 points

9. Differentiate the following functions (*Do not simplify*):

(a) $y = \left(3x + \dfrac{2}{x} - \sqrt{x}\right)(x^4 + 19x^2 - 3x + 2)$.

$[3x + 2x^{-1} - x^{1/2}]'(x^4+19x^2-3x+2) + (3x+\frac{2}{x}-\sqrt{x})[x^4+19x^2-3x+2]'$

$= (3 - 2x^{-2} - \frac{1}{2}x^{-1/2})(x^4+19x^2-3x+2) + (3x+\frac{2}{x}-\sqrt{x})(4x^3+38x-3)$

(b) $h(z) = \dfrac{z^3 + 2z - 1}{z^2 - 3z + 6}$

$$h'(x) = \frac{[z^3+2z-1]'(z^2-3z+6) - (z^3+2z-1)[z^2-3z+6]'}{(z^2-3z+6)^2}$$

$$= \frac{(3z^2+2)(z^2-3z+6) - (z^3+2z-1)(2z-3)}{(z^2-3z+6)^2}$$

10 points

Page 4 Total (20)