Honors Calculus I: Fall 2006
Exam 1

Name:____________________________________

1.[20pts.] Evaluate the following limits. (Use the limit laws and related techniques. Do NOT construct tables using your calculator.)

(a) $\lim_{x\to 2}\left(\frac{1+3x}{1+4x^2}\right)^3$

$$=\left(\lim_{x\to 2}\frac{1+3x}{1+4x^2}\right)^3$$

$$=\left(\frac{7}{17}\right)^3$$

(b) $\lim_{x\to -5^-}\frac{|x+5|}{x+5}$

$$=\lim_{x\to -5^-}\frac{-(x+5)}{x+5}=\lim_{x\to -5^-}-1=-1$$

(c) $\lim_{h\to 0}\frac{h}{\sqrt{1+3h}-1}$

$$=\lim_{h\to 0}\frac{h}{\sqrt{1+3h}-1}\cdot\frac{\sqrt{1+3h}+1}{\sqrt{1+3h}+1}$$

$$=\lim_{h\to 0}\frac{h(\sqrt{1+3h}+1)}{(1+3h)-1}$$

$$=\lim_{h\to 0}\frac{h(\sqrt{1+3h}+1)}{3\cdot h}$$

$$=\lim_{h\to 0}\frac{\sqrt{1+3h}+1}{3}=\frac{2}{3}$$

(d) $\lim_{r\to 9}\frac{\sqrt{r}}{(r-9)^4}$

$\frac{3}{0}$

DNE

2.[10pts.] Let $g(x)=1/x$. Use the definition of the derivative to find $g'(2)$.

$$g'(2)=\lim_{h\to 0}\frac{g(2+h)-g(2)}{h}=\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}=\lim_{h\to 0}\frac{2-(2+h)}{h\cdot 2(2+h)}$$

$$=\lim_{h\to 0}\frac{-h}{h\cdot 2(2+h)}=\lim_{h\to 0}\frac{-1}{2(2+h)}=-\frac{1}{4}$$

3.[12pts.] Define

$$h(x) = \begin{cases} 1/x & \text{for } x < 0, \\ 0 & \text{for } x = 0, \\ 1/x & \text{for } 0 < x \leq 1, \\ \dfrac{x^2 - x}{x - 1} & \text{for } 1 < x. \end{cases}$$

(a) Is h continuous at $x = 0$? Explain why or why not using the definition of continuity.
(b) Is h continuous at $x = 1$? Explain why or why not using the definition of continuity.

a) @ $x=0$ $\lim_{x\to 0^-} h(x) = \lim_{x\to 0^-} \frac{1}{x} = -\infty$ DNE. So $\lim_{x\to 0} h(x)$ dne.

Essential discontinuity

b) @ $x=1$ $\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} \frac{1}{x} = 1$

$\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} \frac{x^2-x}{x-1} = \lim_{x\to 1^+} x = 1$

$h(1) = \frac{1}{1} = 1$

Continuous at $x = 1$.

4.[10pts.] Show that the equation $x^{1/3} = 1 - x$ has at least one solution. Justify your answer using an appropriate theorem.

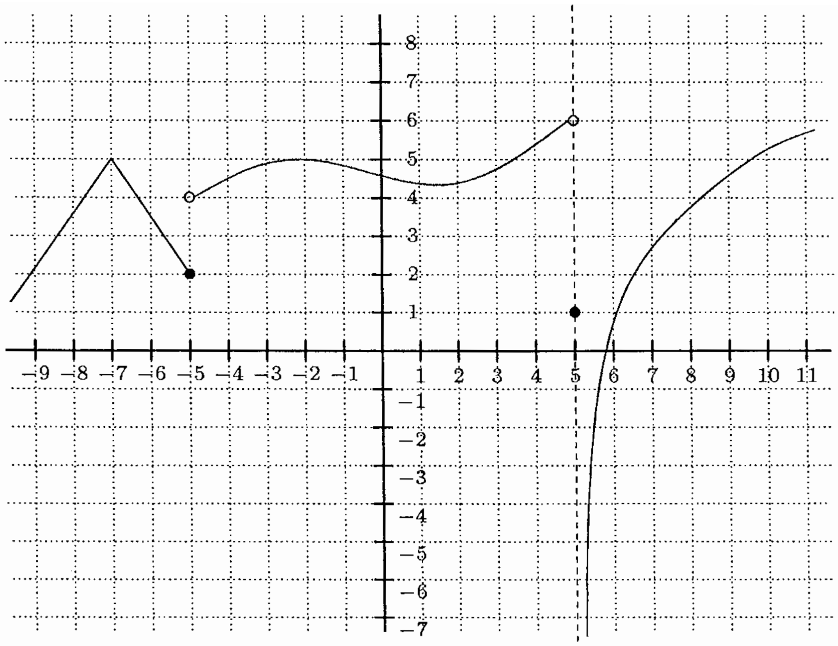
Consider $f = x + x^{1/3} - 1$,

NOTE: f is continuous over $\mathbb{R}$

$f(0) = -1$

$f(1) = 1$

So by I.V.T. f has a root in $(0,1)$. Hence $x^{1/3} = 1-x$ has at least one sol'n.



5.[18pts.] Consider the function f whose graph is given above. Answer the following.

a. $\lim_{x\to -7} f(x) =$ 5

b. $\lim_{x\to -5^+} f(x) =$ 4

c. $\lim_{x\to -5^-} f(x) =$ 2

d. $\lim_{x\to -5} f(x) =$ dne

e. $\lim_{x\to 5^+} f(x) =$ $-\infty$ or dne

f. $\lim_{x\to 5} f(x) =$ dne

6.[10pts.] Let f and g be functions such that $f(3) = -1$, $f'(3) = 2$, $g(3) = 4$, and $g'(3) = 0$.

(a) Find $(fg)'(3)$.

$= f'(3)g(3) + f(3)g'(3)$

$= 2(4) + (-1)(0) = 8$

(b) Find $\left(\dfrac{f}{f-g}\right)'(3)$.

$$= \frac{f'(3)\big(f(3)-g(3)\big) - f(3)\big(f'(3)-g'(3)\big)}{\big(f(3)-g(3)\big)^2}$$

$$= \frac{2(-1-4) - (-1)(2-0)}{(-1-4)^2} = \frac{-10+2}{5^2}$$

$$= \frac{-8}{25}$$

7.[10pts.] Suppose the position of particle moving in a straight line as a function of time is $s(t) = 4t^3 - 15t^2 + 12t - 72$ for all $t \geq 0$. Find the time(s) at which the velocity of the particle is 0.

$$V(t) = s'(t) = 12t^2 - 30t + 12$$
$$= 6(2t^2 - 5t + 2)$$
$$= 6(2t-1)(t-2) = 0$$
$$t = \frac{1}{2}, 2.$$

8.[10pts.] For each of the following functions, find $f'(x)$. Do NOT simplify your answers.

(a) $f(x) = \dfrac{(3-x)(x^3+2) - 8}{\sqrt{x}}$

$$= \frac{3x^3 + 6 - x^4 - 2x - 8}{\sqrt{x}}$$
$$= \frac{-x^4 + 3x^3 - 2x - 2}{\sqrt{x}}$$
$$f = -x^{7/2} + 3x^{5/2} - 2x^{1/2} - 2x^{-1/2}$$
$$f' = -\frac{7}{2}x^{5/2} + \frac{15}{2}x^{3/2} - x^{-1/2} + x^{-3/2}$$

(b) $f(x) = \pi x^{1/3} - x^{2/3}(x^4 + 7) - 21\pi$

$$= \pi x^{1/3} - x^{14/3} - 7x^{2/3} - 21\pi$$
$$f' = \frac{\pi}{3}x^{-2/3} - \frac{14}{3}x^{11/3} - \frac{14}{3}x^{-1/3} + 0$$