CALCULUS I - 3450:221
TEST #1 FALL 05

ROW # _____

NAME Key

CLASS TIME _____

100 Points

Show all your work.

1. Evaluate the following limits:

a. (2 pts) $\lim_{x\to 2}\left(\frac{x-2}{x^2-16}\right)$

$\lim_{x\to 2} \frac{(x-2)}{(x-4)(x+4)} = 0$

b. (6 pts) $\lim_{x\to 1}\left(\frac{2x^2+x-3}{x^2-1}\right)$

$\lim_{x\to 1} \frac{(2x+3)(x-1)}{(x-1)(x+1)} = \frac{5}{2}$

c. (2 pts) $\lim_{x\to 8^+}\left(\frac{|8-x|}{8-x}\right) = -1$

d. (2 pts) $\lim_{x\to 3^-}\left[\frac{x+5}{x^4(x-3)}\right] = -\infty$

e. (6 pts) $\lim_{x\to 1}\left(\frac{\sqrt{x+3}-2}{x-1}\right)$

$\lim_{x\to 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)}$

$= \lim_{x\to 1} \frac{(x+3)-4)}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{(x-1)}{(x-1)(\sqrt{x+3}+2)} = \frac{1}{4}$

f. (2 pts) $\lim_{x\to 3}(\pi^2) = \pi^2$

20 Points

2. (10 pts) **Use the definition of derivative** to find the derivative of $f(x) = \frac{1}{2-x}$

$f'(x) = \lim_{h\to 0} \frac{\frac{1}{2-x-h} - \frac{1}{2-x}}{h}$

$= \lim_{h\to 0} \frac{(2-x) - (2-x-h)}{h(2-x-h)(2-x)}$

$= \lim_{h\to 0} \frac{h}{h(2-x-h)(2-x)} = \frac{1}{(2-x)^2}$

10 Points

30 Points

3. If a ball is thrown into the air with a velocity of 96 ft/sec from the top of a 256 foot tall building its height (in feet) after t seconds is given by $s(t) = 256 + 96t - 16t^2$.

a. (5 pts) Find the maximum height reached by the ball.

5 Points

$s'(t) = 96 - 32t$

max height when $v = 96 - 32t = 0 \Rightarrow t = 3$ s

$s(3) = 256 + 96 \cdot 3 - 16(3)^2 = 400$ ft

b. (5 pts) Find the velocity of the ball when it hits the ground.

5 Points

Hits ground when $256 + 96t - 16t^2 = 0$

$-16(t^2 - 6t - 16) = 0$

$-16(t-8)(t+2) = 0$

$t = 8, -2$

$v(8) = 96 - 32 \cdot 8 = -160$ ft/s

4. a. (5 pts) Find the point on the graph of $f(x) = x^4$ such that the tangent line has a slope of 32 at this point.

5 Points

$f'(x) = 4x^3 = 32$

$x^3 = 8$

$x = 2$

$f(2) = 2^4 = 16$

b. (3 pts) Find the equation of the tangent line to the curve at this point. (Leave your equation in point-slope form.)

3 Points

$\frac{y - 2^4}{x - 2} = 32 \Rightarrow$ ~~$y - 16 = 32(x-2)$~~

$y - 16 = 32(x-2)$

c. (3 pts) Find the equation of the normal line to the curve at this point. (Leave your equation in point-slope form.)

3 Points

normal slope $= -1/32$

$y - 16 = -\frac{1}{32}(x-2)$

21 Points

5. Find the derivatives of the following functions: (**DO NOT** simplify your answers.)

a. (6 pts) $f(x) = \sqrt{7x} + \sqrt[5]{x^2} + \frac{3}{x^2} + 5x - \cos(9)$

$\sqrt{7}x^{1/2} \quad x^{2/5} \quad 3x^{-2}$

$f'(x) = \sqrt{7} \cdot \frac{1}{2} x^{-1/2} + \frac{2}{5} x^{-3/5} - 6x^{-3} + 5$

6 Points

b. (5 pts) $f(x) = \frac{x^2 + 3x - 1}{x^4 + 1}$

$f'(x) = \frac{(x^4+1)(2x+3) - (x^2+3x-1)(4x^3)}{(x^4+1)^2}$

5 Points

c. (5 pts) $f(x) = \frac{x^3 - 2x\sqrt{x} + 5}{x^2}$

$= x - \frac{2\sqrt{x}}{x} + 5x^{-2}$

$= x - 2x^{-1/2} + 5x^{-2}$

5 Points

So $f'(x) = 1 + (-2)(-\frac{1}{2})x^{-3/2} - 10x^{-3}$

d. (6 pts) $f(x) = (x^2 + x + 7)(x^{1/2} + 3x^{-1} - 2x + 1)$

$f'(x) = (x^2 + x + 7)(\frac{1}{2}x^{-1/2} - 3x^{-2} - 2)$

$+ (x^{1/2} + 3x^{-1} - 2x + 1)(2x + 1)$

6 Points

22 Points

6. Use the given graph of f(x) to answer the following questions:

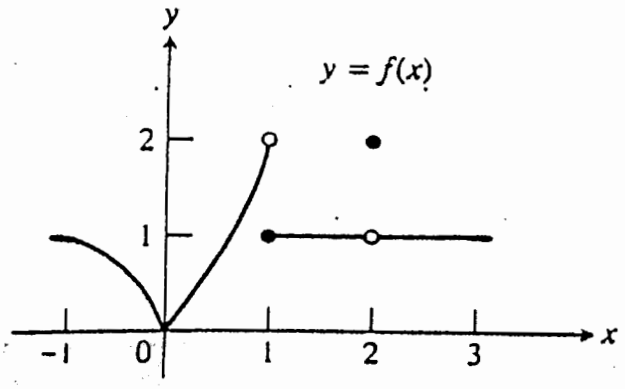
a. (1 pt) $\lim_{x\to 0} f(x) =$ 0

b. (1 pt) $\lim_{x\to 2} f(x) =$ 1

c. (1 pt) $\lim_{x\to 1^-} f(x) =$ 2

d. (1 pt) $\lim_{x\to 1^+} f(x) =$ 1

e. (1 pt) $\lim_{x\to 1} f(x) =$ DNE



f. (6 pts) List the numbers at which f is discontinuous. For each number state the type of discontinuity and the conditions from the definition of continuity which are violated.

Jump discontinuity at $x=1$ because $\lim_{x\to 1} f(x)$ DNE

and so $\lim_{x\to 1} f(x) \neq f(1) = 1$

removable discontinuity at $x=2$ because $f(2) = 2$

and $\lim_{x\to 2} f(x) = 1$. However $\lim_{x\to 2} f(x) \neq f(2)$

6 Points

g. (6 pts) List the numbers at which f is not differentiable and state why f is not differentiable at that number.

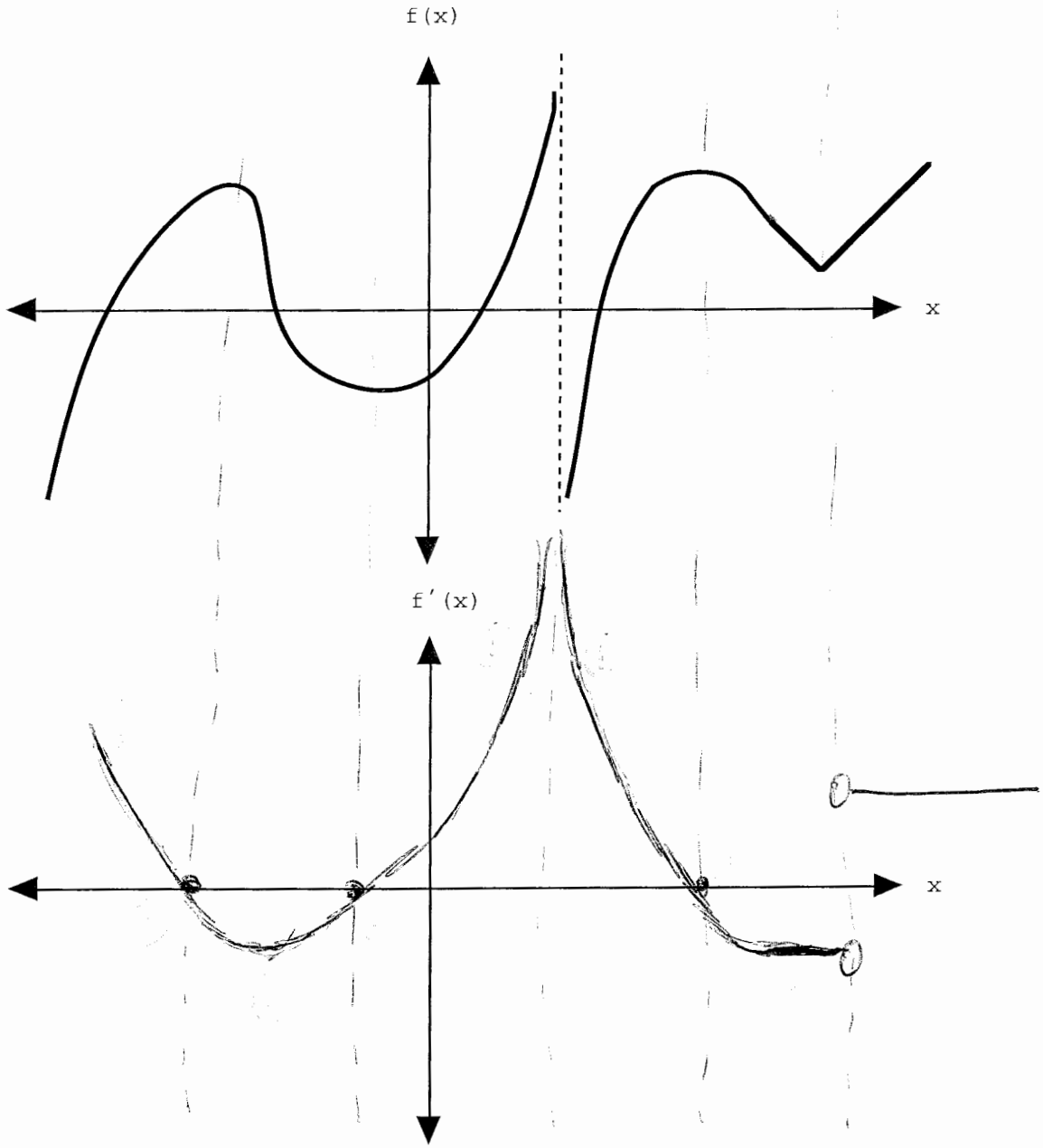
Not dif at 0 because there is a cusp (corner) at that point.

Not dif at $x=1$ or $x=2$ because $f(x)$ is not continuous at these points.

6 Points

17 Points

7. (10 pts) The graph of the function f(x) is given below. Use it to sketch the graph of f '(x).



10 Points