CALCULUS I - 3450:221 CLASS TIME ____________
TEST #1 SPRING 04

NAME ____

100 Points

Show all your work.

1. Evaluate the following limits:

a. (3 pts) $\lim_{x\to 3}\left(\dfrac{x-2}{1+\sqrt{x+6}}\right)$

$$= \frac{1}{1+3}$$

$$= \frac{1}{4}$$

b. (2 pts) $\lim_{x\to \pi}(e)$

$$= e$$

c. (7 pts) $\lim_{x\to 2}\left(\dfrac{4-\sqrt{11x-6}}{x-2}\right)\cdot\dfrac{4+\sqrt{11x-6}}{4+\sqrt{11x-6}}$

$$= \lim_{x\to 2}\left[\frac{16-(11x-6)}{(x-2)(4+\sqrt{11x-6})}\right]$$

$$= \lim_{x\to 2}\left[\frac{22-11x}{(x-2)(4+\sqrt{11x-6})}\right]$$

$$= \lim_{x\to 2}\left[\frac{-11(x-2)}{(x-2)(4+\sqrt{11x-6})}\right]$$

$$= -\frac{11}{8}$$

d. (6 pts) $\lim_{x\to -2}\left(\dfrac{x^2-x-6}{2x^2+3x-2}\right)$

$$= \lim_{x\to -2}\left[\frac{(x-3)(x+2)}{(2x-1)(x+2)}\right]$$

$$= \lim_{x\to -2}\left[\frac{x-3}{2x-1}\right]$$

$$= \frac{-5}{-5}$$

$$= 1$$

e. (2 pts) $\lim_{x\to -3^+}\left[\dfrac{x-4}{x^2(x+3)}\right]$ $\dfrac{-}{(+)(+)}$

$$= -\infty$$

f. (2 pts) $\lim_{x\to 4^-}\left[\dfrac{x-4}{|x-4|}\right]$ $\dfrac{-}{+}$

$$= -1$$

22 Points

2. If $f(x) = \begin{cases} 5 & \text{if } x < -2 \\ x^2 + 1 & \text{if } -2 \le x < 1 \\ 2x^2 + 3 & \text{if } x \ge 1 \end{cases}$ answer the following questions:

a. (4 pts) Is f continuous at x = -2? Explain why or why not using the definition of continuous.

$f(-2) = 5$; $\lim_{x \to -2^+} f(x) = 5$, $\lim_{x \to -2^-} f(x) = 5$ so $\lim_{x \to -2} f(x) = 5$;

$f(-2) = \lim_{x \to -2} f(x)$ so by def. f <u>is</u> cont. at $x = -2$

b. (4 pts) Is f continuous at x = 1? Explain why or why not using the definition of continuous.

$\lim_{x \to 1^-} f(x) = 2$, $\lim_{x \to 1^+} f(x) = 5$ so $\lim_{x \to 1} f(x)$ dne

by def. f is <u>not</u> cont. at $x = 1$

8 Points

3. (9 pts) Use the definition of derivative to find f '(x) if $f(x) = \sqrt{2x + 1}$

$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right] = \lim_{h \to 0} \left[\frac{\sqrt{2x+2h+1} - \sqrt{2x+1}}{h} \right] \cdot \frac{\sqrt{2x+2h+1} + \sqrt{2x+1}}{\sqrt{2x+2h+1} + \sqrt{2x+1}}$$

$$= \lim_{h \to 0} \left[\frac{2x+2h+1 - (2x+1)}{h(\sqrt{2x+2h+1} + \sqrt{2x+1})} \right] = \lim_{h \to 0} \left[\frac{2h}{h(\sqrt{2x+2h+1} + \sqrt{2x+1})} \right]$$

$$= \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

9 Points

4. (4pts) The displacement (in meters) of a particle moving in a straight line is given by

$$s(t) = t^2 - 5t - 1$$

where t is measured in seconds. Find the instantaneous velocity when t = 4 seconds.

$s'(t) = 2t - 5$

$s'(4) = 8 - 5 = 3$ m/s

4 Points

5. Refer to the graph of f(x) to answer the following questions:

a. (1 pt) $\lim_{x \to -2^-} f(x) =$ 3

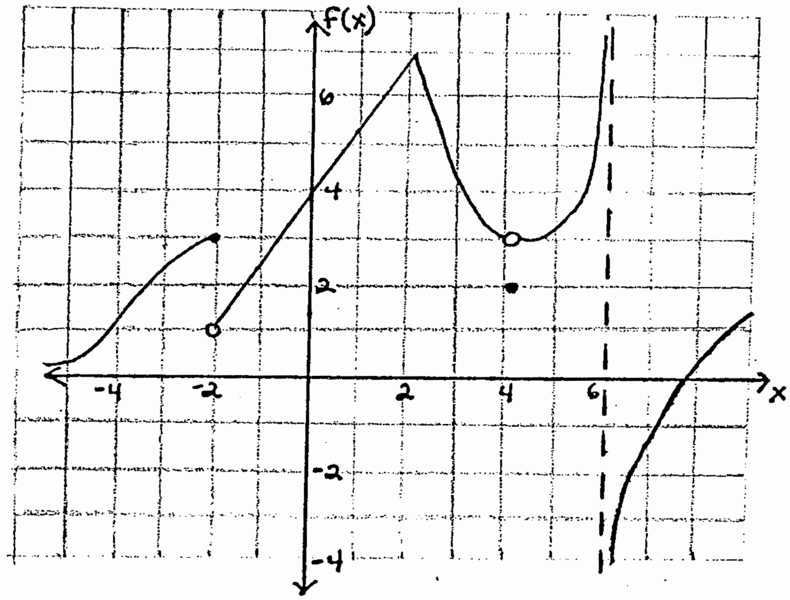
b. (1 pt) $\lim_{x \to -2^+} f(x) =$ 1

c. (1 pt) $\lim_{x \to -2} f(x) =$ dne

d. (1 pt) $\lim_{x \to 2} f(x) =$ 7

e. (1 pt) $\lim_{x \to 4} f(x) =$ 3

f. (1 pt) $\lim_{x \to 6^-} f(x) =$ ∞



g. (6 pts) List the values of x at which f is discontinuous. For each of these values state the condition(s) from the definition of continuity that is (are) violated.

$X = -2$ $\quad \lim_{x \to -2} f(x)$ dne

$X = 4$ $\quad \lim_{x \to 4} f(x) \neq f(4)$

$X = 6$ $\quad \lim_{x \to 6} f(x)$ dne ; $f(6)$ dne

h. (8 pts) State, with reasons, the values of x at which f is not differentiable.

$X = -2$ not continuous

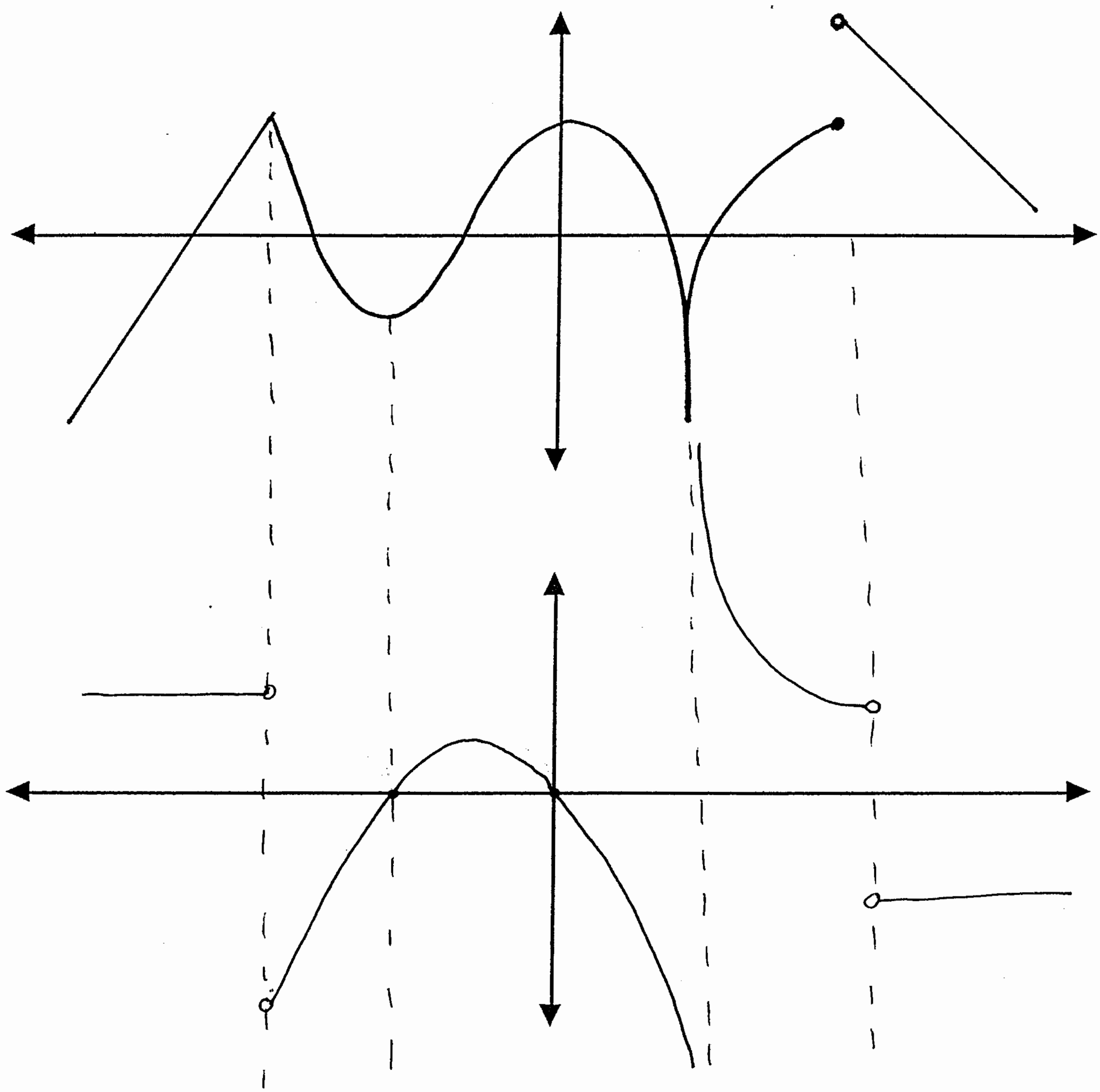
$X = 2$ corner

$X = 4$ not continuous

$X = 6$ not continuous

20 Points

6. (10 pts) The graph of the function f(x) is given below. Use it to sketch the graph of f '(x).



10 Points

7. Find the derivative of the following functions: (**DO NOT** simplify your answer.)

a. (7 pts) $f(x) = 5x + \frac{7}{x^2} + \sqrt[3]{x^2} + 4\pi^2$ $= 5x + 7x^{-2} + x^{2/3} + 4\pi^2$

$f'(x) = 5 - 14x^{-3} + \frac{2}{3}x^{-1/3}$

b. (10 pts) $f(x) = (x^7 + 3x^5 - 2x^4 + 8x^{1/5})(2x^{1/2} - 3x^{-2} + 7x^9)$

$f'(x) = (x^7 + 3x^5 - 2x^4 + 8x^{1/5})(x^{-1/2} + 6x^{-3} + 63x^8)$
$+ (2x^{1/2} - 3x^{-2} + 7x^9)(7x^6 + 15x^4 - 8x^3 + \frac{8}{5}x^{-4/5})$

17 Points

8. (10 pts) Find and **SIMPLIFY** the derivative of:

$$f(x) = \frac{x^2}{x^2 + 2x + 1}$$

AFTER differentiation you may use $x^2 + 2x + 1 = (x + 1)^2$

$$f'(x) = \frac{(x^2+2x+1)(2x) - x^2(2x+2)}{(x^2+2x+1)^2}$$

$$= \frac{2x^3 + 4x^2 + 2x - 2x^3 - 2x^2}{(x^2+2x+1)^2}$$

$$= \frac{2x^2 + 2x}{((x+1)^2)^2}$$

$$= \frac{2x(x+1)}{(x+1)^4}$$

$$= \frac{2x}{(x+1)^3}$$

10 Points