Solutions to MA242 Quiz 1, 09/12/06

1. Is $(3, 4, -2)$ a solution of the following system?

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 7 \\ -2x_1 + 6x_2 + 9x_3 &= 0 \\ -7x_1 + 5x_2 - 3x_3 &= -7 \end{aligned}$$

Solution: To check whether $(3, 4, -2)$ is a solution, set $x_1 = 3$, $x_2 = 4$, and $x_3 = -2$ in the equations and determine whether they are satisfied. You find that the third equation is not, since $-7(3)+5(4)-3(-2) = 5 \neq -7$. Hence, $(3, 4, -2)$ is not a solution to the system.

Note: This is precisely the third practice problem from Section 1.1, see p. 10 of the book.

2. Choose h and k such that the following system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions. (Give separate answers for each part.)

$$\begin{aligned} x_1 + 3x_2 &= 2 \\ 3x_1 + hx_2 &= k \end{aligned}$$

Solution: Row reducing the augmented matrix corresponding to the given system, you find

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & h-9 & k-6 \end{bmatrix}.$$

Hence, (a) the system is inconsistent if $h = 9$ and $k \neq 6$ (since the rightmost, "augmented" column is a pivot column then), (b) the system is consistent and has a unique solution when $h \neq 9$ for *arbitrary* k (since there are no free variables then), and (c) the system has infinitely many solutions if $h = 9$ and $k = 6$ (since x_2 is a free variable then).

Note: This is precisely Exercise 20. from the problem set for Section 1.2, see p. 26 of the book.