CALCULUS III - 3450:223
TEST # 3

NAME Key

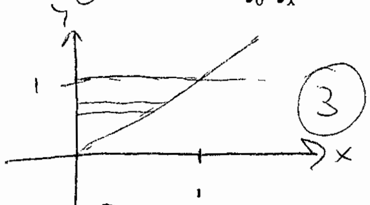
100 Points

Show **ALL** your work. Please circle your final answers.

1. EVALUATE $\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$.

15 Points

$x \le y \le 1$



3

$$\overset{3}{\int_0^1} \overset{3}{\int_0^y} \sin(y^2)\, dx\, dy = \overset{2}{\int_0^1 y \sin(y^2)\, dy}$$

$$= -\frac{1}{2}\cos(y^2)\Big|_0^1 = -\frac{1}{2}\left[\cos 1 - 1\right]$$

2 2

2. EVALUATE $\int_1^2 \int_y^{y^3} e^{x/y}\, dx\, dy$.

10 Points

$$\int_1^2 y e^{\frac{x}{y}}\Big|_y^{y^3} dy \quad (2)$$

$$= \int_1^2 \left[y e^{y^2} - y e\right] dy \quad (2)$$

$$= \left[\frac{1}{2} e^{y^2} - \frac{y^2}{2} e\right]\Big|_1^2 \quad (4)$$

$$= \left(\frac{1}{2}e^4 - 2e\right) - \left(\frac{1}{2}e - \frac{1}{2}e\right)$$

$$= \frac{1}{2}e^4 - 2e \quad (2)$$

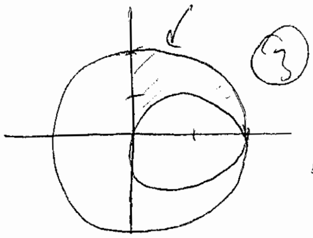
25 Points

3. Set up a double integral in polar coordinates and evaluate it to find the area of the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

$(x-1)^2 + y^2 = 1$

15 Points

$r = 2$



3 3 3

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$$A = \int_0^{\pi/2} \int_{2\cos\theta}^{2} r \, dr \, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2}\left[r^2\right]\Big|_{2\cos\theta}^{2} d\theta = \frac{1}{2}\int_0^{\pi/2}\left[4 - 4\cos^2\theta\right] d\theta$$

1

$$= 2\int_0^{\pi/2} \sin^2\theta \, d\theta = 2\int_0^{\pi/2} \frac{1}{2}\left[1 - \cos 2\theta\right] d\theta$$

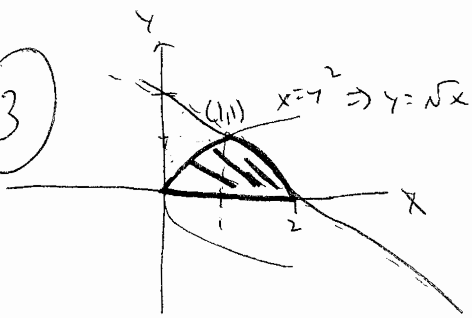
$$= \theta - \frac{1}{2}\sin 2\theta \Big|_0^{\pi/2} = \pi/2$$

4 1

4. Given $\int_0^1 \int_{y^2}^{2-y} f(x, y)\, dx\, dy$, set up an equivalent double integral(s) with the order of integration reversed. DO NOT EVALUATE THE INTEGRAL(S).

15 Points

3



$x = y^2 \Rightarrow y = \sqrt{x}$

$y^2 \le x \le 2-y$

$y^2 = 2-y$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$x = 2-y \Rightarrow y = 2-x$

Split into two integrals

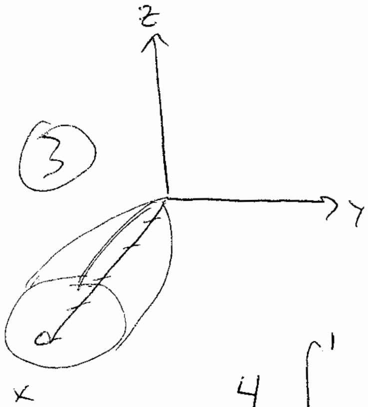
$$\int_0^1 \int_0^{\sqrt{x}} f(x,y)\, dy\, dx + \int_1^2 \int_0^{2-x} f(x,y)\, dy\, dx$$

3 3 3 3

30 points

5. Set up a triple integral(s) to find the volume of the solid bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$. DO NOT EVALUATE THE INTEGRAL(S).

12 Points



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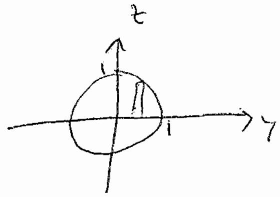


$x = 4 = 4y^2 + 4z^2$

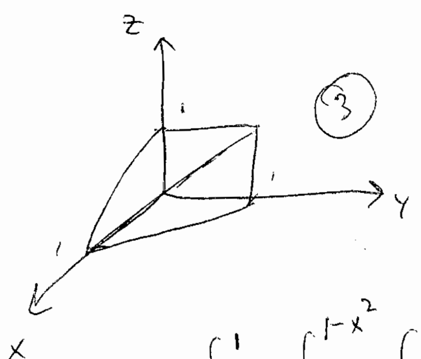
$\Rightarrow 1 = y^2 + z^2$

$$4\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{4y^2+4z^2}^{4} dx\,dz\,dy$$

(2) (5) (4)

6. Use triple integrals to find the z-coordinate of the center of Mass for the solid bounded by $x = 0$, $y = 0$, $z = 0$, $y = 1 - x$, and $z = 1 - x^2$. The density of the region is given by $\delta(x, y, z) = 1 + x$. Set up all of your integrals so that the order of integration is dy dz dx. DO NOT EVALUATE THE INTEGRALS.

18 Points



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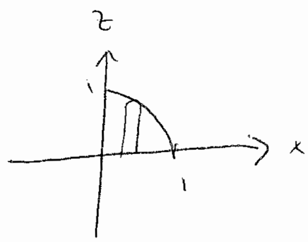

$$M = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} (1+x)\,dy\,dz\,dx$$

(2) (3) (3) (1)

$$M_{xy} = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} (1+x)(z)\,dy\,dz\,dx$$

(3)

$$\bar{z} = \frac{M_{xy}}{M}$$

(3)

30 points

7. Use the technique of Lagrange Multipliers to find the point on the plane 2x - y + z = 3 that is closest to the point (-4, 1, 3).

$$\text{Min } (\text{distance})^2 = (x+4)^2 + (y-1)^2 + (z-3)^2 \quad 3$$

subject to $g(x,y,z) = 2x - y + z - 3 = 0$ 1

$$\nabla f = \lambda \nabla g$$

$$2(x+4) = \lambda \cdot 2 \quad \Rightarrow \quad x = \lambda - 4$$

$$2(y-1) = \lambda(-1) \quad \Rightarrow \quad y = \frac{2-\lambda}{2} = 1 - \lambda/2 \quad 6$$

$$2(z-3) = \lambda(1) \quad \Rightarrow \quad z = 3 + \lambda/2$$

$$2x - y + z - 1 = 0 \Rightarrow 2(\lambda - 4) - (1 - \lambda/2) + (3 + \lambda/2) - 3 = 0$$

$$\cancel{3\lambda - 17/2 = 0}$$

$$\cancel{\Rightarrow \lambda = +17/6}$$

~~So~~ $\cancel{x = 17/6 - 4 = 1/6}$

$$\cancel{y = 1 - 17/12 = -5/12}$$

$$\cancel{z = 3/2 + 17/12 = 35/12}$$

$$\cancel{(1/6,\ -5/12,\ 35/12)}$$

$$3\lambda - 9 = 0 \quad 3$$

$$\Rightarrow \lambda = 3$$

So

$$x = 3 - 4 = -1$$

$$y = 1 - 3/2 = -1/2$$

$$z = 3 + 3/2 = 4\tfrac{1}{2} = 9/2$$

$$(-1, -1/2, 9/2)$$

2

15 Points