Calc III - 3450:223 NAME Key
EXAM III Fall 99 ROW

Show **ALL** your work.

100 Points

1. SET UP BUT DO NOT EVALUATE the integral(s) needed to find the surface area of $z = 4 + x^2 + y^2$ that lies under $z = 13$.

10 Points

$$SA = \iint \sqrt{1 + f_x^2 + f_y^2}\, dA$$

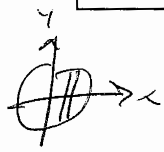
$f = 4 + x^2 + y^2$

$f_x = 2x$

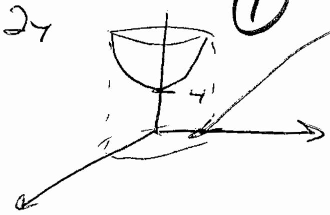
$f_y = 2y$

intersection $13 = 4 + x^2 + y^2$

$9 = x^2 + y^2$



1



$$SA = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \sqrt{1 + 4x^2 + 4y^2}\, dy\, dx$$

2 3 4

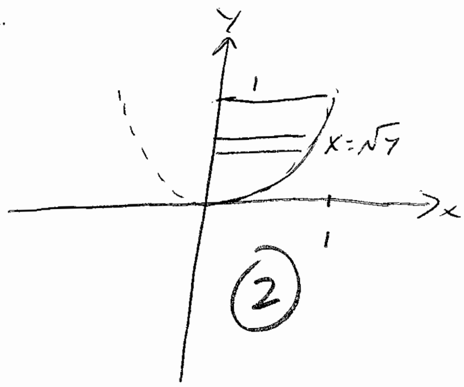
2. Evaluate $\int_0^1 \int_{x^2}^1 4x\sin(y^2)\, dydx$.

12 Points

Switch order

$x^2 \le y \le 1$

$0 \le x \le 1$



2

2 4

$$\int_0^1 \int_0^{\sqrt{y}} 4x \sin y^2\, dx\, dy$$

$$= \int_0^1 2x^2 \sin y^2 \Big|_0^{\sqrt{y}} dy \quad 1$$

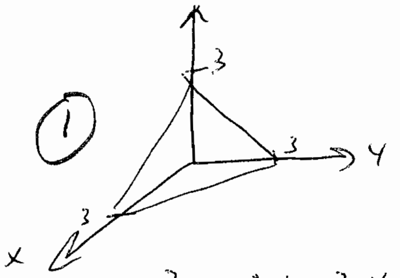
$$= \int_0^1 2y \sin(y^2)\, dy$$

$$= -\cos(y^2) \Big|_0^1 = -\cos 1 + 1$$

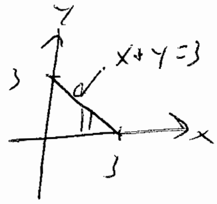
2 1

22 Points

3. SET UP BUT DO NOT EVALUATE a triple integral to find the volume of the region bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 3$.



1



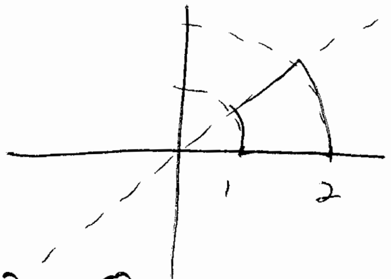
10 Points

$$V = \int_0^3 \int_0^{3-x} \int_0^{3-x-y} dz\, dy\, dx$$

3 3 3

4. Evaluate $\iint_R 3y \, dA$ where R is the region in the first quadrant bounded by $y = 0$, $y = x$, $x^2 + y^2 = 1$, and $x^2 + y^2 = 4$.

12 Points



switch to polar

2 2 2 2

$$\int_0^{\pi/4} \int_1^2 3(r\sin\theta)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/4} r^3 \sin\theta \Big|_1^2 d\theta$$

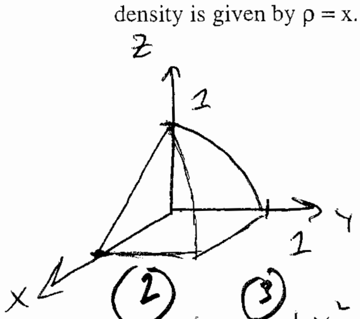
$$= \int_0^{\pi/4} (8-1)\sin\theta \, d\theta$$ 2

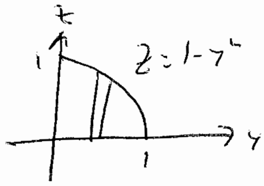
$$= -7\cos\theta \Big|_0^{\pi/4} = -7\left(\frac{\sqrt{2}}{2} - 1\right)$$

2

22 Points

5. SET UP BUT DO NOT EVALUATE integrals to find the y–coordinate of the center of mass of the solid bounded by $z = 1 - y^2$, $x = 0$, $y = 0$, $z = 0$, and $x + z = 1$, if the density is given by $\rho = x$.





12 Points

$$M = \int_0^1 \int_0^{1-y^2} \int_0^{1-z} x \, dx \, dz \, dy \quad \text{or} \int_0^1 \int^{1-x} \int_0^{\sqrt{1-z}} x \, dy \, dz \, dx$$

(2) (3) (3)

$$M_{xz} = \int_0^1 \int_0^{1-y^2} \int_0^{1-z} x \cdot y \, dx \, dz \, dy \quad \text{or} \int_0^1 \int^{1-x} \int_0^{\sqrt{1-z}} xy \, dy \, dz \, dx$$

(3)

$$\bar{y} = \frac{M_{xz}}{M}$$

(1)

6. Evaluate $\int_0^1 \int_x^{2x} \int_0^{x+y} 6xy \, dz dy dx$.

12 Points

$$\int_0^1 \int_x^{2x} 6xyz \Big|_0^{x+y} dy \, dx$$

$$= \int_0^1 \int_x^{2x} (6x^2y + 6xy^2) \, dy \, dx \quad (3)$$

$$= \int_0^1 (3x^2y^2 + 2xy^3) \Big|_x^{2x} dx \quad (4)$$

$$= \int_0^1 (12x^4 + 16x^4 - 3x^4 - 2x^4) \, dx$$

$$= \int_0^1 23x^4 \, dx \quad (4)$$

$$= \frac{23x^5}{5}\Big|_0^1 = \frac{23}{5} \quad (1)$$

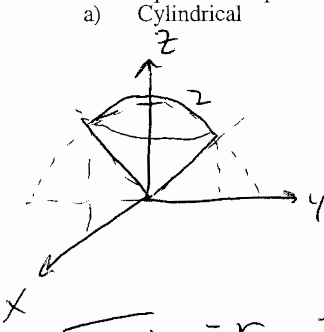
24 Points

7. A solid Q is bounded above by $x^2 + y^2 + z^2 = 4$ and bounded below by $z = \sqrt{x^2 + y^2}$.

Consider the integral $\iiint_Q e^{-(x^2+y^2+z^2)}\, dV$. SET UP BUT DO NOT EVALUATE

equivalent expressions for this integral in cylindrical and spherical coordinates.

a) Cylindrical



10 Points

1 2 3 2

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} e^{-(r^2+z^2)}\, r\,dz\,dr\,d\theta$$

2

$z = \sqrt{x^2+y^2} = r$

$r^2 + z^2 = 4$ intersect $\Rightarrow$ $r^2 + r^2 = 2r^2 = 4 \Rightarrow r^2 = 2,\ r = \sqrt{2}$

b) Spherical

10 Points

cone

$\rho\cos\phi = \rho\sin\phi$

so $\phi = \pi/4$

3 2

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} e^{-\rho^2}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

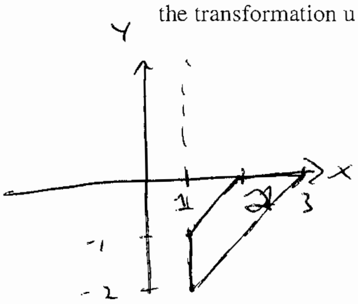
1 2 2

8. SET UP BUT DO NOT EVALUATE an integral equivalent to $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$,

where R is the region bounded by $x = 1$, $y = 0$, $x - y = 2$, and $x - y = 3$, by using

the transformation $u = x + y$, and $v = x - y$.

$y = x - 2$ $y = x - 3$



12 Points

$u = x + y$

$v = x - y$

$u + v = 2x \Rightarrow x = \frac{1}{2}(u+v)$

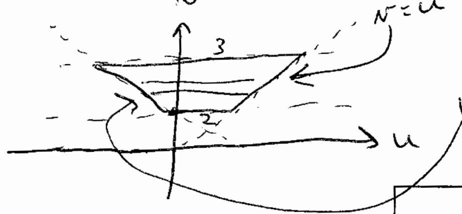
$u - v = 2y \Rightarrow y = \frac{1}{2}(u-v)$

$x = 1 \Rightarrow u + v = 2$

$y = 0 \Rightarrow u - v = 0$

$v = u$ $v = 2 - u$

or $u = 2 - v$



$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2}$$

$$\int_2^3\int_{2-v}^{v} \cos\left(\frac{-v}{u}\right)\cdot\frac{1}{2}\,du\,dv$$

2 4 2 4

32 Points