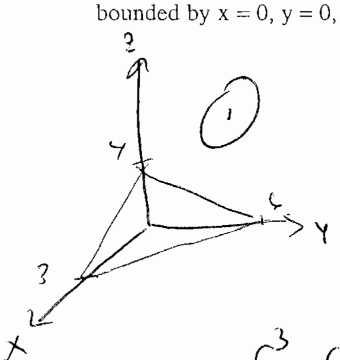
Calc III - 3450:223
EXAM # 3 Fall97

NAME Key
ROW

Show **ALL** your work.

100 Points

1. SET UP BUT DO NOT EVALUATE a triple integral to find the volume of the region bounded by $x = 0$, $y = 0$, $z = 0$, and $4x + 2y + 3z = 12$.



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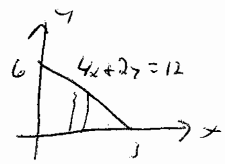


10 Points

$$V = \int_0^3 \int_0^{6-2x} \int_0^{\frac{12-4x-2y}{3}} dz\, dy\, dx$$

3 3 3

2. Evaluate $\int_0^1 \int_0^z \int_0^y xyz \, dxdydz$.

12 Points

$$\int_0^1 \int_0^z \frac{x^2 y z}{2} \Big|_0^y dy\, dz \quad 4$$

$$= \int_0^1 \int_0^z \frac{y^3 z}{2} dy\, dz$$

$$= \int_0^1 \frac{y^4 z}{8} \Big|_0^z dz \quad 4$$

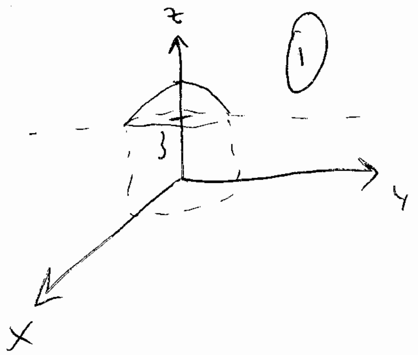
$$= \int_0^1 \frac{z^5}{8} dz \quad 3$$

$$= \frac{z^6}{48} \Big|_0^1 = 1/48 \quad 1$$

22 Points

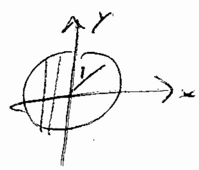
3. SET UP BUT DO NOT EVALUATE the integral(s) needed to find the surface area of $z = 4 - x^2 - y^2$ that lies above $z = 3$.

10 Points



$3 = 4 - x^2 - y^2$

$\Rightarrow x^2 + y^2 = 1$

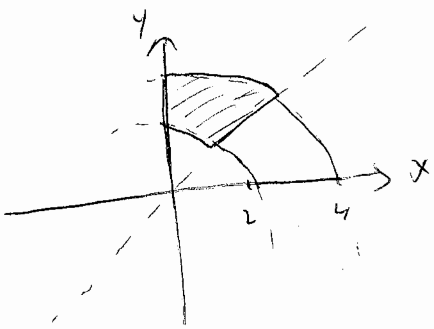


$$SA = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{(-2x)^2 + (-2y)^2 + 1} \; dy\,dx$$

2 3 4

4. Evaluate $\iint_R x \, dA$ where R is the region bounded by $x = 0$, $y = x$, $x^2 + y^2 = 4$, and $x^2 + y^2 = 16$.

12 Points



Use polar

2 2 2 2

$$\int_{\pi/4}^{\pi/2} \int_{2}^{4} r\cos\theta \cdot r \, dr \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \frac{r^3}{3} \cos\theta \Big|_2^4 \, d\theta$$

2

$$= \int_{\pi/4}^{\pi/2} \left(\frac{64}{3} - \frac{8}{3}\right) \cos\theta \, d\theta = 8$$

$$\frac{56}{3} \sin\theta \Big|_{\pi/4}^{\pi/2} = \frac{56}{3}\left[1 - \frac{\sqrt{2}}{2}\right]$$

2

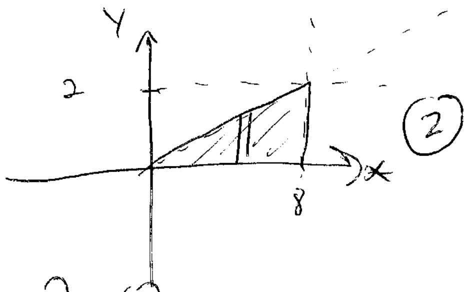
22 Points

5. Evaluate $\int_0^2 \int_{4y}^8 e^{x^2}\,dxdy$.

$4y \le x \le 8$

$0 \le y \le 2$

12 Points



2 4

$$\int_0^8 \int_0^{\frac{x}{4}} e^{x^2}\,dy\,dx = \int_0^8 y e^{x^2} \Big|_0^{x/4} dx$$ 1

$$= \int_0^8 \frac{1}{4} x e^{x^2}\,dx$$ 2

$$= \frac{1}{8} e^{x^2} \Big|_0^8 = \frac{1}{8}\left[e^{64} - 1\right]$$ 1

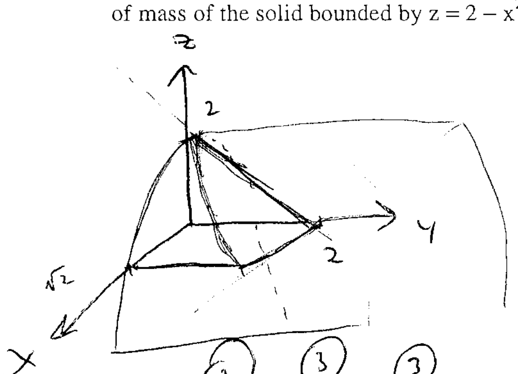
6. SET UP BUT DO NOT EVALUATE integrals to find the y–coordinate of the center of mass of the solid bounded by $z = 2 - x^2$, $y = 0$, and ~~[illegible]~~.

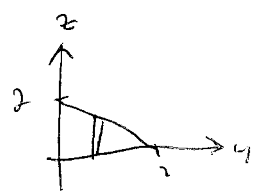
$x = 0$

$z = 0$

$y + z = 2$

12 Points





2 3 3

$$\text{Mass} = \int_0^2 \int_0^{2-y} \int_0^{\sqrt{2-z}} dx\,dz\,dy \quad \text{or} \quad \int_0^{\sqrt{2}} \int_0^{2-x^2} \int_0^{2-z} dy\,dz\,dx$$

$$M_{xz} = \int_0^2 \int_0^{2-y} \int_0^{\sqrt{2-z}} y\,dx\,dz\,dy \quad \text{or} \quad \int_0^{\sqrt{2}} \int_0^{2-x^2} \int_0^{2-z} y\,dy\,dz\,dx$$

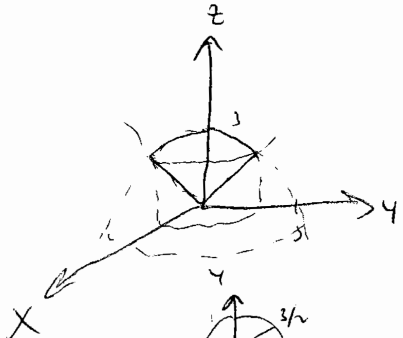
3

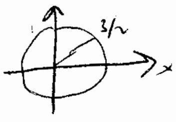
$$\bar{y} = \frac{M_{xz}}{M}$$ 1

24 Points

7. A solid Q is bounded above by $x^2 + y^2 + z^2 = 9$ (Sphere) and bounded below by $z = \sqrt{3(x^2 + y^2)}$ (Cone).
Consider the integral $\iiint_Q e^{-(x^2+y^2+z^2)}\, dV$. SET UP BUT DO NOT EVALUATE equivalent expressions for this integral in cylindrical and spherical coordinates.

a) Cylindrical





$x^2+y^2+3(x^2+y^2)=9$

$x^2+y^2=\frac{9}{4}$

$$\int_0^{2\pi}\int_0^{3/2}\int_{r\sqrt{3}}^{\sqrt{9-r^2}} e^{-(r^2+z^2)}\, r\, dz\, dr\, d\theta$$

(1) (2) (3) (2) (2) (2)

10 Points

b) Spherical

$\rho = 3$

$\rho\cos\phi = \sqrt{3}\, r = \sqrt{3}\,\rho\sin\phi$

$\cos\phi = \sqrt{3}\sin\phi$

$\tan\phi = \frac{1}{\sqrt{3}}$

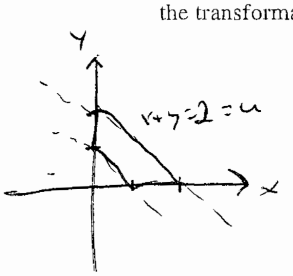
$\Rightarrow \phi = \pi/6$

$$\int_0^{2\pi}\int_0^{\pi/6}\int_0^{3} e^{-\rho^2}\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

(1) (3) (2) (2) (2)

10 Points

8. SET UP BUT DO NOT EVALUATE an integral equivalent to $\iint_R \sin\left(\frac{y-x}{y+x}\right) dA$,
where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$, and $x + y = 2$, by using the transformation $u = x + y$, and $v = x - y$.



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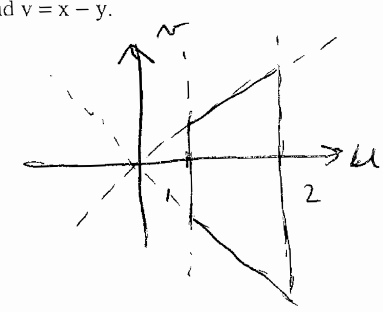

12 Points

$x=0 \Rightarrow u=y$
$v=-y$
$\Rightarrow u=-v$

$y=0 \Rightarrow u=x$
$v=x$
$\Rightarrow u=v$

$u = x+y$

$v = x-y$

$\frac{u+v}{2} = x$

$\frac{u-v}{2} = y$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac12 & \frac12 \\ \frac12 & -\frac12 \end{vmatrix} = -\frac12$$ (4)

$$\int_1^2\int_{-u}^{u} \sin\left(\frac{-v}{u}\right)\frac12\, dv\, du$$

(2) (4) (2)

32 Points