CALCULUS III - 3450:223
TEST # 2

NAME Key

100 Points

Show **ALL** your work. Please circle your final answers.

1. Find all the first partial derivatives of the function $f(x, y, z) = e^{x^2y} + \sin(yz)$.

10 Points

$f_x = 2xy e^{x^2 y}$ (3)

$f_y = x^2 e^{x^2 y} + z\cos(yz)$ (4)

(2) (2)

$f_z = y\cos(yz)$ (3)

2. Find the domain and range and sketch two level curves of $z = \sqrt{9 - x^2 - 3y^2}$.

15 Points

(2) (3)

range $z \geq 0$, $z \leq 3$ so $0 \leq z \leq 3$ (5)

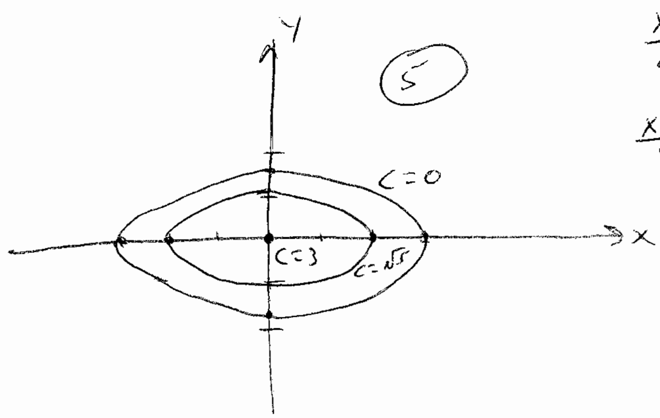
domain $9 - x^2 - 3y^2 \geq 0 \Rightarrow x^2 + 3y^2 \leq 9$ (5)

Level curves are ellipses $x^2 + 3y^2 = 9 - c^2$

(5)

$\frac{x^2}{9} + \frac{y^2}{3} = 1$, $c = 0$

$\frac{x^2}{4} + \frac{3}{4}y^2 = 1$, $c = \sqrt{5}$



25 Points

3. Given $f(x, y) = \dfrac{4x^2 + 7y^2}{7x^2 - 2y^2}$, discuss $\lim\limits_{(x, y) \to (0, 0)} f(x, y)$ and discuss the continuity of $f(x, y)$.

10 Points

$$\lim_{(x,0)\to(0,0)} \frac{4x^2}{7x^2} = 4/7 \quad 3$$

Limit DNE 6

$$\lim_{(0,y)\to(0,0)} \frac{7y^2}{-2y^2} = -7/2 \quad 3$$

Continuous everywhere except at the lines

$$7x^2 - 2y^2 = 0 \Rightarrow y = \pm\sqrt{\tfrac{7}{2}}\, x$$

4

4. Classify the critical points of $3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

15 Points

3 $f_x = 6xy - 6x = 6x(y-1) = 0 \Rightarrow x = 0$ or $y = 1$

$f_y = 3x^2 + 3y^2 - 6y = 0$

$x = 0 \Rightarrow 3y^2 - 6y = 0 = 3y(y-2) \Rightarrow y = 0, 2$

$\cancel{x} = 1 \Rightarrow 3x^2 + 3 - 6 = 3x^2 - 3 = 3(x^2 - 1) = 0 \Rightarrow x = \pm 1$

Critical points are $(0,0)$, $(0,2)$, $(1,1)$, $(-1,+1)$ 4

$f_{xx} = 6y - 6$

$f_{yy} = 6y - 6$

$f_{xy} = 6x$

$$g(x,y) = f_{xx}f_{yy} - (f_{xy})^2 = (6y-6)^2 - 36x^2$$

$(0,0)$	$(0,2)$	$(1,1)$	$(-1,1)$
$f_{xx} = -6 < 0$	$f_{xx} = 12 - 6 = 6 > 0$	$f_{xx} = 0$	$f_{xx} = 0$
$g(0,0) = 36 > 0$	$g(0,2) = 36 > 0$	$g(1,1) = -36 < 0$	$g(-1,1) = -36 < 0$
local max	local min	saddle	saddle
2	2	2	2

25 Points

5. Given $v = x + \sin(y - z) + w^2$, where $x = s^2$, $y = \ln(s + t)$, $z = s\cos(t)$, and $w = s\sin(t)$, use the chain rule to find $\frac{\partial v}{\partial s}$ and $\frac{\partial v}{\partial t}$ in terms of s and t.

10 Points

$$\frac{\partial v}{\partial s} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial s} + \frac{\partial v}{\partial w}\frac{\partial w}{\partial s} \quad (2)$$

$$= 1\cdot 2s + \cos(y-z)\frac{1}{s+t} + -\cos(y-z)\cos t + 2w\sin t \quad (3)$$

(5)

$$= 2s + \cos[\ln(s+t) - s\cos t]\frac{1}{s+t} - \cos[\ln(s+t) - s\cos t]\cos t + 2s\sin t\sin t$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial v}{\partial w}\frac{\partial w}{\partial t} \quad (2)$$

$$= 1\cdot 0 + \cos(y-z)\frac{1}{s+t} + -\cos(y-z)(-s\sin t) + 2w(s\cos t) \quad (5)$$

(3)

$$= 0 + \cos[\ln(s+t) - s\cos t)\frac{1}{s+t} - \cos[\ln(s+t) - s\cos t](-s\sin t) + 2\cdot s\sin t\, s\cos t$$

6a. The temperature T at any point in a steel ball, centered at the origin, is given by $T(x, y, z) = 360(x^2 + y^2 + z^2)^{-1/2}$. Find the rate of change of T at (1, 2, 2) in the direction toward the point (2, 1, 3).

10 Points

$$\vec{u} = (2-1)\hat{i} + (1-2)\hat{j} + (3-2)\hat{k} = \hat{i} - \hat{j} + \hat{k}$$

$$\hat{u} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} \quad (2)$$

$$(6)\; D_{\hat{u}}T = \nabla T\cdot\hat{u} = -360(x^2+y^2+z^2)^{-3/2}[x\hat{i} + y\hat{j} + z\hat{k}]\cdot\frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{3}}\Big|_{(1,2,2)}$$

(3) (5)

$$= \frac{-360(9)^{-3/2}[1 + 2(-1) + 2(1)]}{\sqrt{3}} = \frac{-360}{9\cdot 3\sqrt{3}} = \frac{-40}{3\sqrt{3}} = -7.698 \quad (2)$$

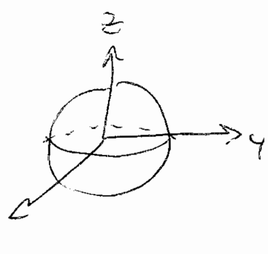
6b. Sketch a level surface of the temperature function.

3 Points

$$c = 360(x^2+y^2+z^2)^{-1/2} \Rightarrow$$

$$x^2 + y^2 + z^2 = \left(\frac{360}{c}\right)^2$$

sphere centered at origin



6c. Find the rate of change of T if one travels along a curve lying on this level surface.

2 Points

On the surface T is a constant so

$D_{\hat{u}}T = 0$ for all $\hat{u}$ tangent to the surface.

25 Points

7. Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ at which the tangent plane is parallel to the plane $3x - y + 3z = 1$.

15 Points

$\vec{n}_{\text{tangent plane}} = \nabla F = 2x\hat{i} + 4y\hat{j} + 6z\hat{k}$ (2)

$\vec{n}$ to given plane $= 3\hat{i} - 1\hat{j} + 3\hat{k}$ (2)

parallel $\Rightarrow$ $2x = 3c \Rightarrow x = \frac{3}{2}c$

$4y = -c \Rightarrow y = -\frac{1}{4}c$ (8)

$6z = 3c \Rightarrow z = \frac{1}{2}c$

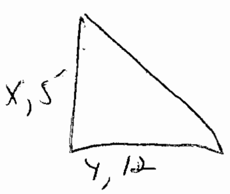
(x, y, z) on the ellipsoid $\Rightarrow \left(\frac{3}{2}c\right)^2 + 2\left(-\frac{1}{4}c\right)^2 + 3\left(\frac{1}{2}c\right)^2 = 1$

or $c^2\left[\frac{9}{4} + \frac{1}{8} + \frac{3}{4}\right] = 1$ (3)

$c^2\left(\frac{25}{8}\right) = 1 \Rightarrow c = \pm\sqrt{\frac{8}{25}} = \frac{2\sqrt{2}}{5}$ points $\left(\pm\frac{3\sqrt{2}}{5}, \mp\frac{\sqrt{2}}{10}, \pm\frac{\sqrt{2}}{5}\right)$

8a. The two legs of a right triangle are measured as 5 m and 12m, respectively, with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle.

7Points



$A = \frac{1}{2}xy$ (2)

$dA = \frac{1}{2}[y\,dx + x\,dy]$ (3)

$= \frac{1}{2}[1200(0.2) + 500(0.2)]$ (2)

$= \frac{1}{2}[340] = 170\text{cm}^2$

8b. Use differentials to estimate the maximum error in the calculated length of the hypotenuse.

8 Points

$h = \sqrt{x^2 + y^2}$ (2)

$dh = \frac{1}{2}(x^2+y^2)^{-1/2}[2x\,dx + 2y\,dy]$ (4)

$= \frac{1}{\cancel{2}}(5^2+12^2)^{-1/2}[\cancel{2}\cdot 5\cdot 0.2 + \cancel{2}\cdot 12\cdot 0.2]$

$= \frac{1}{13}[3.4] = \frac{3.4}{13} = 0.2615\text{ cm}$ (2)

30 Points