Calc III - 3450:223	NAME Key	64 Points
EXAM #2 F06	ROW	

Show **ALL** your work.

1. For the function $z = x^3y^2$

a) Calculate the normal vector to this surface at the point (x, y, z) = (1, 3, 9). **4 Points**

$$F = z - x^3y^2$$

$$\nabla F = \vec{n} = \langle -3x^2y^2, -2x^3y, 1 \rangle \quad (3)$$

$$= \langle -27, -6, 1 \rangle \quad (1)$$

b) Find the equation of the tangent plane at the point (1, 3, 9). **4 Points**

$$\langle -27, -6, 1 \rangle \cdot \langle x-1, y-3, z-9 \rangle = 0 \quad (3)$$

$$-27x - 6y + z + (27 + 18 - 9) = 0$$

$$-27x - 6y + z + 36 = 0 \quad (1)$$

c) Find the direction of maximal increase of the function at the point (x, y) = (1, 3). **4 Points**

$$\text{Max increase} = \nabla f = \langle 3x^2y^2, 2x^3y \rangle \quad (3)$$

$$= \langle 27, 6 \rangle$$

(1)

d) Find the directional derivative of f(x, y) in the direction of $-4\mathbf{i} + 3\mathbf{j}$ at the point (x, y) = (1, 3). **6 Points**

$$D_{\hat{u}} f = \langle 27, 6 \rangle \cdot \frac{\langle -4, 3 \rangle}{\sqrt{16+9}} \quad (1) \quad (2)$$

(2)

$$= \langle 27, 6 \rangle \cdot \frac{\langle -4, 3 \rangle}{5} = \frac{-108 + 18}{5}$$

$$= \frac{-90}{5} = -18 \quad (1)$$

18 Points

2. Find the domain and range of $z = \sqrt{x^2 + y^2 - 4}$, and sketch and label two level curves of this function.

Domain is (x, y) such that $x^2 + y^2 \geq 4$ (2)

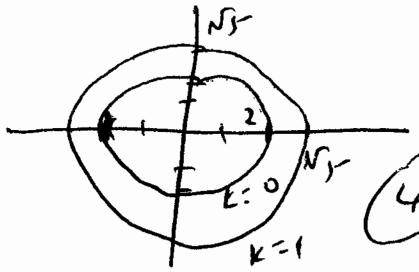
Range is all $z \geq 0$ (2)

8 Points

$k = \sqrt{x^2 + y^2 - 4}$

so $x^2 + y^2 = k^2 + 4$

circle, radius greater or equal to 2



(4)

3. Use Lagrange multipliers to find the extrema of $f(x, y, z) = 6x + 4y + 12z$ subject to the constraint $x^2 + 2y^2 + 3z^2 = \frac{23}{4}$.

$W = 6x + 4y + 12z - \lambda\left(x^2 + 2y^2 + 3z^2 - \frac{23}{4}\right)$

$W_x = 6 - 2x\lambda = 0 \Rightarrow x = 3/\lambda$

$W_y = 4 - 4y\lambda = 0$ $\quad y = 1/\lambda$ (3)

$W_z = 12 - 6z\lambda = 0$ $\quad z = 2/\lambda$

$W_\lambda \Rightarrow x^2 + 2y^2 + 3z^2 = 23/4$

(3) $\frac{9}{\lambda^2} + 2 \cdot \frac{1}{\lambda^2} + 3 \cdot \frac{4}{\lambda^2} = \frac{23}{\lambda^2} = \frac{23}{4}$

so $\lambda^2 = 4 \Rightarrow \lambda = \pm 2$ (2)

10 Points

at $(3/2, 1/2, 1)$

$f = 6 \cdot 3/2 + 4 \cdot 1/2 + 12 = 9 + 2 + 12 = 23$ max

at $(-3/2, -1/2, -1)$

$f = -23$ min (2)

4. Given $z = f(x, y)$ with $x = g(t)$ and $y = h(t)$, find $\frac{dz}{dt}$ when $t = 3$, if $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(7, 2) = 9$, $f_x(2, 7) = 6$, $f_y(7, 2) = 3$ and $f_y(2, 7) = -8$.

$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

$= f_x \cdot g'(t) + f_y h'(t)$ (4)

$x = g(3) = 2$

$y = h(3) = 7$

$= f_x(2, 7)\, g'(3) + f_y(2, 7)\, h'(3)$

$= 6 \cdot 5 + -8 \cdot (-4) = 30 + 32 = 62$

(4)

8 Points
26 Points

5. Find and then use the Second Partials Test to classify the critical points of $f(x, y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y$.

10 Points

1 $f_x = 2x - 2y = 0 \Rightarrow x = y$

1 $f_y = -2x + y^2 - 3 = y^2 - 2y - 3 = (y-3)(y+1) = 0$

so $y = 3, -1$

crit points $(-1,-1)$ and $(3,3)$ 2

$f_{xx} = 2 > 0$

$f_{yy} = 2y$

$f_{xy} = -2$

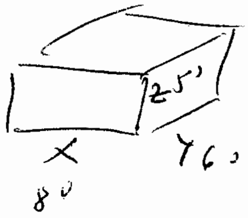
2 1

$D = f_{xx} f_{yy} - (f_{xy})^2 = 2 \cdot (2 \cdot -1) - (-2)^2 = -8 < 0$ saddle at $(-1,-1)$

$D = 2 \cdot 2 \cdot 3 - (-2)^2 = 8 > 0$ with $f_{xx} > 0$ so min at $(3,3)$

2 1

6. The dimensions of a closed rectangular box are measured as 80 cm, 60 cm and 50 cm, respectively with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.



$SA = 2xy + 2xz + 2yz$ 3

10 Points

$dSA = (2y + 2z)\,dx + 2(x+z)\,dy + 2(x+y)\,dz$ 3

$= 2(60+50)\,0.2 + 2(80+50)\,0.2 + 2(80+60)\,0.2$

$= 220 \cdot 0.2 + 260 \cdot 0.2 + 280 \cdot 0.2$ 3

$= (760) \cdot 0.2 = 152 \text{ cm}^2$

1

20 Points

36 Points

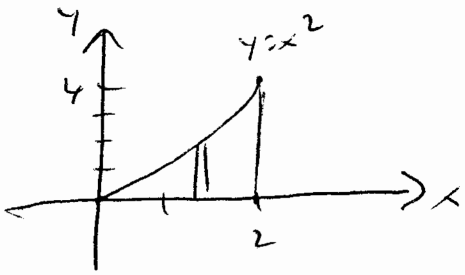
Calc III - 3450:223
EXAM #2 F06
Show **ALL** your work.

NAME Key
ROW

1. Evaluate $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dxdy.$ switch order of int

10 Points

$\sqrt{y} \le x \le 2 \qquad 0 \le y \le 4$ (2)



(1)

(2)

$$\int_0^2 \int_0^{x^2} e^{x^3}\, dy\, dx$$

$$= \int_0^2 y e^{x^3} \Big|_0^{x^2} dx$$

$$= \int_0^2 x^2 e^{x^3} dx \quad (2)$$

$$= \frac{1}{3} e^{x^3} \Big|_0^2 = \frac{1}{3}\left[e^8 - 1\right]$$

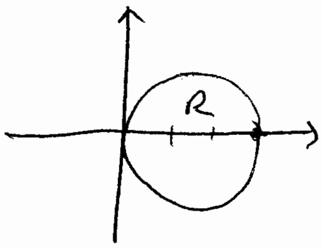
(2) (1)

2. SET UP BUT DO NOT EVALUATE the integral in polar coordinates that represents the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 9$ which lies inside the cylinder $x^2 + y^2 = 3x$ and above the xy plane.

8 Points

$r^2 = 3r\cos\theta$

$r = 3\cos\theta$



$$z = \sqrt{9 - x^2 - y^2}$$

$$z_x = \frac{-2x \cdot \frac{1}{2}}{\sqrt{9 - x^2 - y^2}}$$

$$z_y = \frac{-y}{\sqrt{9 - x^2 - y^2}}$$

$$SA = \iint_R \sqrt{1 + \frac{x^2}{9-x^2-y^2} + \frac{y^2}{9-x^2-y^2}}\, dA = \int_{-\pi/2}^{\pi/2} \int_0^{3\cos\theta} \sqrt{1 + \frac{r^2}{9-r^2}}\, r\, dr\, d\theta$$

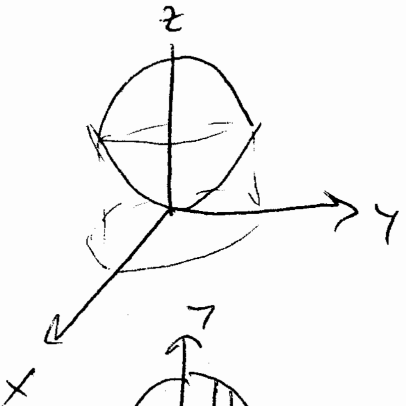
(1)

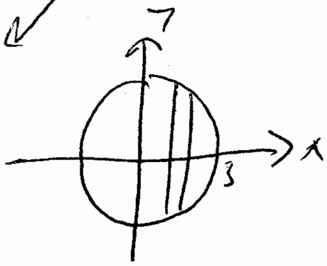
(2) (2) (3)

18 Points

3. SET UP BUT DO NOT EVALUATE an integral representing the volume of the region bounded between $z = 2x^2 + 2y^2$ and $z = 36 - 2x^2 - 2y^2$

a) in cartesian coordinates.





$$2x^2+2y^2 = 36-2x^2-2y^2$$
$$4(x^2+y^2) = 36$$
$$x^2+y^2 = 9$$

6 Points

$$V = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left[(36-2x^2-2y^2)-(2x^2+2y^2)\right] dy\,dx$$

2 2 2

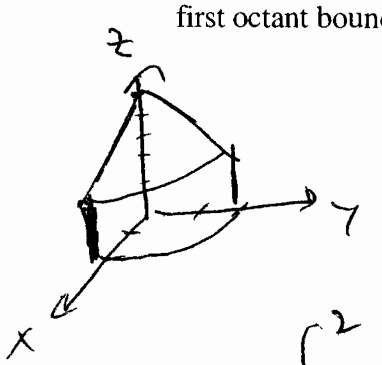
b) in polar coordinates.

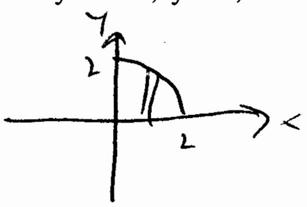
$$\int_0^{2\pi}\int_0^{3} \left[36-4r^2\right] r\,dr\,d\theta$$

2 2 2

6 Points

4. SET UP BUT DO NOT EVALUATE an integral to find the volume of the solid in the first octant bounded by $x = 0$, $y = 0$, $z = 0$, $x + y + z = 5$ and inside $x^2 + y^2 = 4$.





6 Points

$$V = \int_0^{2}\int_0^{\sqrt{4-x^2}} \left[5-x-y\right] dy\,dx$$

2 2 2

18 Points