Calc III - 3450:223
EXAM #2 Fall 99
Show **ALL** your work.

NAME Key
ROW

100 Points

1. Find all the first partial derivatives of $f(x, y, z) = \sin(y^3 e^{x^2yz})$. You do not have to simplify your answers.

10 Points

$f_x = \cos(y^3 e^{x^2yz}) \cdot y^3 e^{x^2yz} \cdot 2xyz$ 3

$f_y = \cos(y^3 e^{x^2yz}) \cdot [3y^2 e^{x^2yz} + y^3 e^{x^2yz} \cdot x^2 z]$ 4

$f_z = \cos(y^3 e^{x^2yz}) \cdot [y^3 e^{x^2yz} \cdot x^2 y]$ 3

2. Classify the critical points of $f(x, y) = x^3 - 3xy + y^3 - 5$.

12 Points

2 $f_x = 3x^2 - 3y = 0 \Rightarrow y = x^2$

$f_y = -3x + 3y^2 = 0 = -3x + 3x^4$

$= -3x(1 - x^3) = 0 \Rightarrow x = 0, 1$

$(0, 0)$ and $(1, 1)$ are critical pts 4

$f_{xx} = 6x$ $D = 6x \cdot 6y - (-3)^2$ 2

$f_{yy} = 6y$

$f_{xy} = -3$

	$(0,0)$	$(1,1)$
f_{xx}	0	6
f_{yy}	0	6
f_{xy}	-3	-3
D	$-9 < 0$	$36 - 9 > 0$
	Saddle	local min

2 2

22 Points

3. Given $f(x, y) = \frac{y^2 - 3x^2}{x^2 - 3y^2}$, determine $\lim_{(x,y)\to(0,0)} f(x, y)$ and determine where f(x, y) is continuous.

10 Points

$\lim_{(0,y)\to(0,0)} \frac{y^2}{-3y^2} = -\frac{1}{3}$ $\lim_{(x,0)\to(0,0)} \frac{-3x^2}{x^2} = -3$ 3

3 So limit DNE

Cont everywhere except when $x^2 - 3y^2 = 0$ 4

4. If $z = x + f(y^2 - x^2)$, where f is a differentiable function, then determine $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x}$.

10 Points

$\frac{\partial z}{\partial x} = 1 + f'(y^2 - x^2) \cdot (-2x)$ 4

$\frac{\partial z}{\partial y} = f'(y^2 - x^2) \cdot 2y$ 4

$x z_y + y z_x = f'(y^2 - x^2) \cdot 2xy + y + f'(y^2 - x^2)(-2xy)$

$= y$ 2

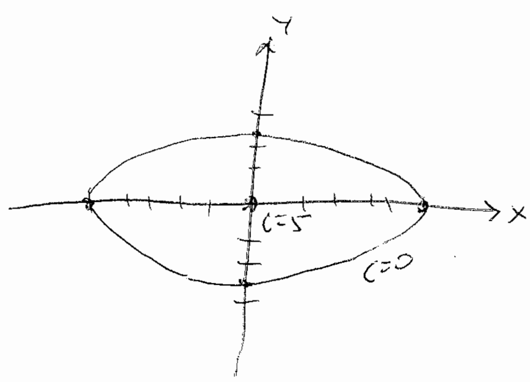
5. Find the domain, range, and sketch two level curves of $z = \sqrt{25 - x^2 - 5y^2}$.

10 Points

Domain $25 - x^2 - 5y^2 \geq 0 \Rightarrow x^2 + 5y^2 \leq 25$ 2

Range $0 \leq z \leq 5$ 3

$c = \sqrt{\quad} \Rightarrow x^2 + 5y^2 = 25 - c^2$ ellipse



5

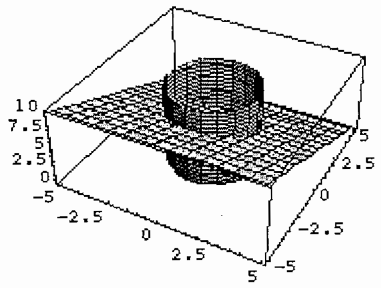
30 Points

6. The plane $y + z = 3$ intersects the cylinder $x^2 + y^2 = 5$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point (1, 2, 1).

12 Points

Need pt $(1,2,1)$

direction = $\perp$ to both normals



$F = y+z-3=0 \Rightarrow \nabla F = \langle 0, 1, 1 \rangle$ 3

$G = x^2+y^2-5=0 \Rightarrow \nabla G = \langle 2x, 2y, 0 \rangle = \langle 2, 4, 0 \rangle$ 3

direction $= \nabla F \times \nabla G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 4 & 0 \end{vmatrix} = -4\hat{i} + 2\hat{j} - 2\hat{k}$ 3

line $\Rightarrow$
$x = 1 - 4t$
$y = 2 + 2t$
$z = 1 - 2t$
3

7. Find the equation of the plane tangent to $x^2 + y^2 = z^2 + 2xy - 4xz + 4$ at the point (1, 0, 1).

Need pt $(1,0,1)$ and normal

$F = x^2+y^2-z^2-2xy+4xz-4=0$ 3

$\vec{n} = \nabla F = \langle 2x-2y+4z, 2y-2x, -2z+4x \rangle$
$= \langle 6, -2, 2 \rangle$ 6

12 Points

plane $\langle 6, -2, 2 \rangle \cdot \langle x-1, y, z-1 \rangle = 0$
3

24 Points

8. The temperature at any point in the metal ball $(x-3)^2+(y-2)^2+(z-5)^2 \le 9$ is given by $T(x, y, z) = xz\sqrt{y}$. Find the rate of change of T as you move from the point (5, 4, 6) on the surface of the ball towards the center of the ball.

12 Points

Ball Center $= (3, 2, 5)$ 4

$\vec{u} = \dfrac{-(5,4,6)+(3,2,5)}{\text{magnitude}} = \dfrac{\langle -2,-2,-1\rangle}{\sqrt{9}} = \dfrac{-\langle 2,2,1\rangle}{3}$

$\nabla T = \langle z\sqrt{y},\ xz\cdot\frac{1}{2}y^{-1/2},\ x\sqrt{y}\rangle = \langle 6\sqrt{4},\ 5\cdot 6\cdot\frac{1}{2}\frac{1}{\sqrt{4}},\ 5\sqrt{4}\rangle$

4 $= \langle 12,\ 15/2,\ 10\rangle$

$\nabla T\cdot\vec{u} = \langle 12,\ 15/2,\ 10\rangle\cdot\dfrac{-\langle 2,2,1\rangle}{3} = \dfrac{24+15+10}{3} = \dfrac{-49}{3}$

3 1

9. Use Lagrange multipliers to find the maximum and minimum temperature on the surface of the sphere $x^2+y^2+z^2=9$ if the temperature at any point on the surface is given by $T(x, y, z) = 2x + y - 2z$.

12 Points

max/min $2x+y-2z$ subject to $x^2+y^2+z^2=9$

Solve
$2 = \lambda 2x \Rightarrow x = \frac{1}{\lambda}$
3 $1 = \lambda 2y$ $\quad y = \frac{1}{2}\frac{1}{\lambda}$ 3
$-2 = \lambda 2z$ $\quad z = -1/\lambda$

$x^2+y^2+z^2 = \dfrac{1}{\lambda^2} + \dfrac{1}{4\lambda^2} + \dfrac{1}{\lambda^2} = \dfrac{9}{4\lambda^2} = 9$

$\Rightarrow 4\lambda^2 = 1$ or $\lambda^2 = \frac{1}{4} \Rightarrow \lambda = \pm\frac{1}{2}$ 3

So $x = \pm 2$
$y = \pm 1$
$z = \mp 2$ 3

Max $= 2\cdot 2 + 1 - 2(-2) = 9$
min $= 2\cdot(-2) - 1 - 2(+2) = -9$

24 Points