Calc III - 3450:223
EXAM # 2 Fall97

NAME Key
ROW

Show **ALL** your work.

100 Points

1. Find all the first partial derivatives of $f(x, y, z) = x \ln(xyz) - \cos(y^3 e^{yz})$. You do not have to simplify your answers.

10 Points

$$f_x = \ln(xyz) + \frac{x \cdot yz}{xyz} \quad 3$$

$$f_y = \frac{\cancel{x} \cdot x\cancel{z}}{\cancel{x}y\cancel{z}} \quad 1 \quad + \sin(y^3 e^{yz})\left[3y^2 e^{yz} + y^3 e^{yz} \cdot z\right] \quad 3$$

$$f_z = \frac{\cancel{x} \cdot x\cancel{y}}{\cancel{x}\cancel{y}z} \quad 1 \quad + \sin(y^3 e^{yz})\left[y^3 e^{yz} \cdot y\right] \quad 2$$

2. Classify the critical points of $f(x, y) = \frac{x^2}{4} - \frac{x^3}{3} + 3xy^2 - 6y^2 + 19$.

12 Points

$f_x = \frac{x}{2} - x^2 + 3y^2 = 0$ 2

$f_y = 6xy - 12y = 6y(x-2) = 0 \Rightarrow y = 0$ or $x = 2$ 2

If $y = 0$ then $f_x = \frac{x}{2} - x^2 = 0$ so $x = 0, \frac{1}{2}$ 2

If $x = 2$ then $f_x = 1 - 4 + 3y^2 = 3y^2 - 3 = 0 \Rightarrow y = \pm 1$ 2

So points are	$(0,0)$	$(\frac{1}{2},0)$	$(2,1)$	$(2,-1)$
$f_{xx} = \frac{1}{2} - 2x$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{7}{2}$	$-\frac{7}{2}$
$f_{yy} = 6x - 12$	-12	-9	0	0
$f_{xy} = 6y$	0	0	6	-6
$D = f_{xx} f_{yy} - (f_{xy})^2$	-6	$\frac{9}{2}$	-36	-36
	saddle 1	max 1	saddle 1	saddle 1

22 Points

3. Find the domain, range, and sketch two level curves of $z = \ln(x^2 - y)$.

10 Points

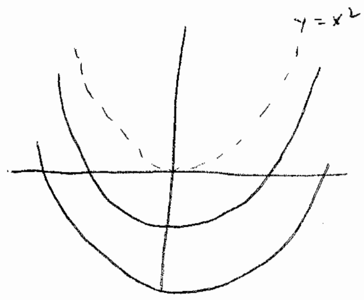
Domain need $x^2 - y > 0 \Rightarrow y < x^2$ (3)

Range of $\ln$ is all $\mathbb{R}$ (2)

Level curves $c = \ln(x^2 - y) \Rightarrow e^c = x^2 - y$

$\Rightarrow y = x^2 - e^c$

Parabolas



(5)

4. Given $f(x, y) = \dfrac{4xy}{5x^2 - 3y^2}$, determine $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ and determine where $f(x, y)$ is continuous.

10 Points

$\lim\limits_{(0,y)\to(0,0)} \dfrac{0}{-3y^2} = 0$ (3) DNE

let $y = x \Rightarrow \lim\limits_{(x,x)\to(0,0)} \dfrac{4x^2}{5x^2 - 3x^2} = 2$ (3)

Cont everywhere except on $5x^2 - 3y^2 = 0$ (4)

5. If $z = y + f(x^2 - y^2)$, where f is a differentiable function, then determine $y\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y}$.

$\dfrac{\partial z}{\partial x} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial x} = f' \cdot 2x$ (4)

$\dfrac{\partial z}{\partial y} = 1 + \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial y} = 1 + f' \cdot (-2y)$ (4)

10 Points

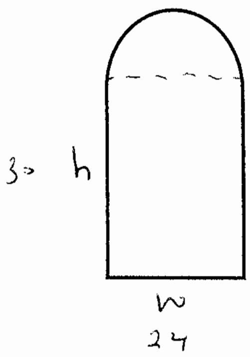
$y z_x + x z_y = y \cdot \cancel{f' 2x} + \cancel{x} \cdot [1 - \cancel{f' 2y}]$

$= x$ (2)

30 Points

6. As shown below a window has the shape of a rectangle surmounted by a semi–circle. The height and width of the rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the window.

12 Points



$$A = h\cdot w + \frac{1}{2}\pi\left(\frac{w}{2}\right)^2$$

$$= h\cdot w + \frac{1}{8}\pi w^2 \quad (4)$$

$$dA = w\cdot dh + \left[h + \frac{1}{4}\pi w\right]dw \quad (4)$$

$$= 24(0.1) + \left[30 + \frac{1}{4}\pi\cdot 24\right]0.1 \quad (4)$$

$$= (5.4 + 0.6\pi)\ \text{cm}^2$$

$$7.28$$

7. The temperature at any point on the surface of the sphere $x^2 + (y-1)^2 + (z-1)^2 = 11$ is given by $T(x, y, z) = z^2e^{x\sqrt{y}}$. Find the rate of change of T in the direction normal to the sphere at the point (1, 4, 2) on the sphere.

12 Points

$$\vec{n} = 2x\,\hat{i} + 2(y-1)\,\hat{j} + 2(z-1)\,\hat{k}$$

$$= \langle 2, 6, 2\rangle \quad (3)$$

$$\vec{u} = \frac{\langle 2, 6, 2\rangle}{\sqrt{4+36+4}} = \frac{\langle 1, 3, 1\rangle}{\sqrt{11}} \quad (2)$$

$$D_{\vec{u}}T = \nabla T\cdot\vec{u} = \left\langle z^2 y^{1/2} e^{x\sqrt{y}},\ z^2 e^{x\sqrt{y}}\cdot\frac{1}{2}x y^{-1/2},\ 2z\,e^{x\sqrt{y}}\right\rangle\cdot\vec{u} \quad (3)\ (3)$$

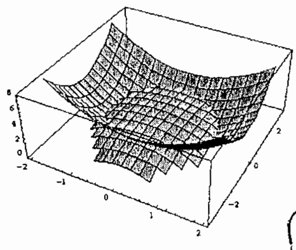
$$= \left\langle 4\cdot 2e^2,\ 4\cdot e^2\cdot\frac{1}{2}\cdot\frac{1}{2},\ 4e^2\right\rangle\cdot\vec{u}$$

$$= e^2\langle 8, 1, 4\rangle\cdot\frac{\langle 1, 3, 1\rangle}{\sqrt{11}} = \frac{15e^2}{\sqrt{11}} \quad (1)$$

24 Points

8. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $(-1, 1, 2)$.

12 Points



Need point $(-1, 1, 2)$
and direction

Direction is $\perp$ to the normals

$F: z - x^2 - y^2 = 0 \Rightarrow \nabla F = \langle -2x, -2y, 1\rangle = \langle 2, -2, 1\rangle$ (3)

$G: 4x^2 + y^2 + z^2 - 9 = 0 \Rightarrow \nabla G = \langle 8x, 2y, 2z\rangle = \langle -8, 2, 4\rangle$ (3)

Need $\nabla F \times \nabla G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ -8 & 2 & 4 \end{vmatrix} =$ [illegible] $-10\hat{i} - 16\hat{j} - 12\hat{k}$ (3)

So line
$x = -1 + -10t$
$y = 1 + -16t$ (3)
$z = 2 + -12t$

9. Use Lagrange multipliers to find three numbers x, y, z, whose sum is 30 and whose product is a maximum.

12 Points

$w = xyz - \lambda(x + y + z - 30)$

(4)

$w_x = yz - \lambda = 0 \qquad \lambda = yz$
$\qquad \Rightarrow x = y$ (6)
$w_y = xz - \lambda = 0 \qquad \lambda = xz$
$\qquad \Rightarrow y = z$
$w_z = xy - \lambda = 0 \qquad \lambda = xy$

$w_\lambda = x + y + z - 30 = 0$

$x = y = z \Rightarrow 3x - 30 = 0$ (2)

$\Rightarrow x = y = z = 10$

24 Points