16 1/2

Calc III - 3450:223
EXAM #1 F06

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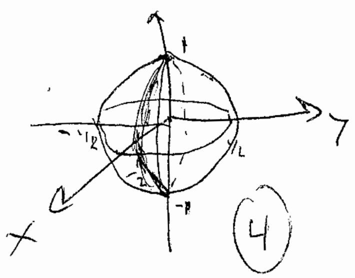
ROW

100 Points

Show **ALL** your work.

1. Identify and sketch the surface $4x^2 + 4y^2 + z^2 = 1$, then convert this to cylindrical coordinates.

ellipsoid 2



$4(x^2+y^2)+z^2=1$

$4r^2+z^2=1$

4

10 Points

2. Find the equation of the plane containting the points (3, −1, 2), (8, 2, 4) and (−1, −2, −3).

P Q R

$\vec{PQ} = \langle 5, 3, 2\rangle$ 2 $\vec{PR} = \langle -4, -1, -5\rangle$ 2

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 3 & 2 \\ -4 & -1 & -5 \end{vmatrix} = (-15+2)\hat{i} - \hat{j}(-25+8) + \hat{k}(-5+12)$$

2

$= \langle -13, 17, 7\rangle$ 3

15 Points

$\langle -13, 17, 7\rangle \cdot \langle x-3, y+1, z-2\rangle = 0$

C

$-13x + 17y + 7z + 39 + 17 - 14 = 0$

$-13x + 17y + 7z + 42 = 0$

3. Convert $\rho^2[\sin^2\phi - 4\cos^2\phi] = 1$ to rectangular coordinates and identify the surface.

2 2

$r^2 - 4z^2 = 1$

$x^2 + y^2 - 4z^2 = 1$ 1

7 Points

Hyperboloid 1 sheet 2

32 Points

4. The position of a projectile is given by $\mathbf{r}(t) = 2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + 5t\,\mathbf{k}$. Find the following

a) The projectile's velocity, $\mathbf{v}$ and acceleration $\mathbf{a}$

$$\vec{v} = \langle 2\cos t, -2\sin t, 5\rangle \quad (2)$$

$$\vec{a} = \langle -2\sin t, -2\cos t, 0\rangle \quad (1)$$

3 Points

b) The speed of the projectile

$$\|\vec{v}\| = \sqrt{4\cos^2 t + 4\sin^2 t + 25} \quad (1)$$

$$= \sqrt{29} \quad (1)$$

2 Points

c) The unit tangent vector, $\mathbf{T}$

$$\vec{T} = \frac{\vec{r}\,'}{\|\vec{r}\,'\|} = \frac{\langle 2\cos t, -2\sin t, 5\rangle}{\sqrt{29}} \quad (1) \quad (1)$$

2 Points

d) The unit normal vector, $\mathbf{N}$

$$\vec{N} = \frac{\vec{T}\,'}{\|\vec{T}\,'\|} = \frac{\frac{1}{\sqrt{29}}\langle -2\sin t, -2\cos t, 0\rangle}{\frac{1}{\sqrt{29}}\sqrt{4\sin^2 t + 4\cos^2 t}} \quad (1) \quad (1)$$

$$= \langle -\sin t, -\cos t, 0\rangle$$

2 Points

e) The curvature, κ

$$\kappa = \frac{\|\vec{T}\,'\|}{\|\vec{r}\,'\|} = \frac{2}{\sqrt{29}} \Big/ \sqrt{29} = \frac{2}{29}$$

(1) (2)

3 Points

f) The tangential and normal components of acceleration, a_T and a_N.

$$a_T = \vec{a}\cdot\vec{T} = \frac{\langle -2\sin t, -2\cos t, 0\rangle \cdot \langle 2\cos t, -2\sin t, 5\rangle}{\sqrt{29}} \quad (3)$$

$$= \frac{-4\sin t\cos t + 4\cos t\sin t}{\sqrt{29}} = 0$$

6 Points

$$a_N = \vec{a}\cdot\vec{N} = \langle -2\sin t, -2\cos t, 0\rangle \cdot \langle -\sin t, -\cos t, 0\rangle$$

$$= +2\sin^2 t + 2\cos^2 t = 2 \quad (3)$$

18 Points

5. Find the vector projection of $\mathbf{v} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ onto the vector $\mathbf{w} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

$$\text{comp}_{\vec{w}}\vec{v} = \frac{\langle 1, 6, -2\rangle \cdot \langle 2, -3, 1\rangle}{\sqrt{4+9+1}} = \frac{2-18-2}{\sqrt{14}} = \frac{-18}{\sqrt{14}}$$

1 3 1

10 Points

$$\text{proj}_{\vec{w}}\vec{v} = \frac{-18}{\sqrt{14}} \frac{\langle 2, -3, 1\rangle}{\sqrt{14}} = -\frac{9}{7}\langle 2, -3, 1\rangle$$

2 3

6. Find the angle between the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

$$\cos\theta = \frac{\langle 1, -2, 1\rangle \cdot \langle 2, 1, 1\rangle}{\sqrt{1+4+1}\ \sqrt{4+1+1}} = \frac{2-2+1}{6} = \frac{1}{6}$$

3 3 1

8 Points

so $\theta = \cos^{-1}(1/6)$

1

7a. Given $\mathbf{r}'(t) = 12\mathbf{i} + 12\sqrt{t}\,\mathbf{j} + 6t\,\mathbf{k}$ and $\mathbf{r}(1) = <0, 0, 0>$, find the equation of the line tangent to $\mathbf{r}(t)$ at $t = 4$.

$$\vec{r}(t) = (12t + c_1)\hat{i} + (8t^{3/2} + c_2)\hat{j} + (3t^2 + c_3)\hat{k}$$

$$\vec{r}(1) = (12 + c_1)\hat{i} + (8 + c_2)\hat{j} + (3 + c_3)\hat{k} = \langle 0, 0, 0\rangle$$

$$c_1 = -12 \qquad c_2 = -8 \qquad c_3 = -3$$

10 Points

$$\vec{r}(t) = (12t - 12)\hat{i} + (8t^{3/2} - 8)\hat{j} + (3t^2 - 3)\hat{k} \quad 6$$

$$\vec{r}\,'(4) = \langle 12, 24, 24\rangle \qquad \vec{r}(4) = \langle 36, 56, 45\rangle \quad 1$$

tan line
$$x = 36 + 12t$$
$$y = 56 + 24t$$
$$z = 45 + 24t$$
3

7b. Find the arclength of $\mathbf{r}(t)$ from $t = 0$ to $t = 1$.

$$S = \int_0^1 \sqrt{144 + 144t + 36t^2}\, dt = \int_0^1 \|\vec{r}\,'\|\, dt \quad 2$$

7 Points

$$= \int_0^1 \sqrt{36(t^2 + 4t + 4)}\, dt$$

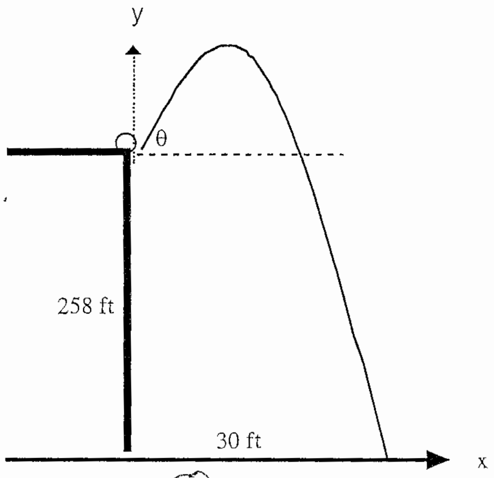
$$= \int_0^1 6(t+2)\, dt \quad 2$$

$$= 6\left[\frac{t^2}{2} + 2t\right]\Big|_0^1 = 6\left[\frac{1}{2} + 2\right] = 6 \cdot \frac{5}{2} = 15$$

2 1

35 Points

8. A ball is thrown at an angle of $\theta = 45^{o}$ from the top of a 258ft tall building as shown in the figure. If the ball lands on the ground 30ft away from the building, what was the initial speed of the ball.



3

2

$\vec{r}(t) = (v_0 \cos\alpha)t\,\hat{i} + [-16t^2 + (v_0 \sin\alpha)t + 258]\hat{j}$

$(v_0 \cos\alpha)t = (v_0 \cos 45)t = (v_0 \frac{\sqrt{2}}{2})t = 30$

$\Rightarrow v_0 = \frac{30 \cdot 2}{t\sqrt{2}} = \frac{60}{t\sqrt{2}}$ 2

$-16t^2 + \left(\frac{60}{t\sqrt{2}} \sin 45\right)t + 258 = 0$ when hits ground

$-16t^2 + \frac{60}{t\sqrt{2}} \cdot \frac{\sqrt{2}}{2} t + 258 = 0$ 3

$-16t^2 + 288 = 0$

$t^2 = \frac{288}{16} = 18$

so $t = \sqrt{18} = 3\sqrt{2}$ 3

Thus $v_0 = \frac{60}{3\sqrt{2}\cdot\sqrt{2}} = \frac{60}{6} = 10s$ 2

15 Points