Calc III - 3450:223
EXAM #1 Fall 99
Show **ALL** your work.

NAME Key
ROW

100 Points

1. Given vectors **a**, **b**, and **c**, show using magnitudes and/or the dot product and/or the cross product how you would

a) decide if **a** and **b** are parallel

$\vec{a} \times \vec{b} = \vec{0} \Rightarrow$ parallel

(1) (1)

2 Points

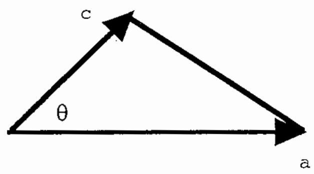
b) decide if **a** and **b** are orthogonal

$\vec{a} \cdot \vec{b} = 0 \Rightarrow$ orthogonal

(1) (1)

2 Points

c) calculate the area of the triangle determined by **a** and **c**



$\frac{1}{2}\|\vec{a} \times \vec{c}\|$

(1) (1)

2 Points

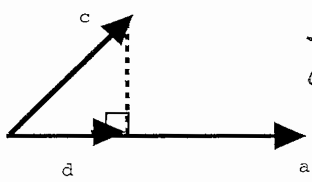
d) calculate the angle θ

$\cos\theta = \dfrac{\vec{a} \cdot \vec{c}}{|\vec{a}||\vec{c}|}$

(1) (1)

2 Points

e) find the vector **d**



$\vec{d} = \left(\vec{c} \cdot \dfrac{\vec{a}}{|\vec{a}|}\right) \dfrac{\vec{a}}{|\vec{a}|}$

(2) (1)

3 Points

f) determine a vector parallel to **a** which is 5 units long.

$5 \cdot \dfrac{\vec{a}}{|\vec{a}|}$

(1) (2)

3 Points

2. Identify the given surface and convert its equation to cylindrical coordinates: $x^2 + y^2 + z^2 = 2x$.

(3) sphere $C(1,0,0)$ $R = 1$

8 Points

$r^2 + z^2 = 2r\cos\theta$

(2) (1) (2)

$(x^2 - 2x + 1) + y^2 + z^2 = 1$

$C(1, 0, 0)$ Radius $= 1$

22 Points

3. The position of a projectile is given by $\mathbf{r}(t) = 6\,\mathbf{i} + t\,\mathbf{j} + \ln[\cos(t)]\,\mathbf{k}$, $\cos(t) > 0$. Find the following:

a) The projectile's velocity, **v**

$$\vec{r}\,' = \vec{v} = 0\,\hat{i} + \hat{j} + \frac{-\sin t}{\cos t}\,\hat{k}$$

(1) (1)

2 Points

b) The speed of the projectile

$$s = |\vec{r}\,'| = \sqrt{1 + \tan^2 t} = \sec t$$

(1) (1)

2 Points

c) The unit tangent vector, **T**

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle 0, 1, -\tan t\rangle}{\sec t} = \langle 0, \cos t, -\sin t\rangle$$

(1) (1)

2 Points

d) The unit normal vector, **N**

(1) (1)

$$\vec{N} = \frac{\vec{T}\,'}{|\vec{T}\,'|} = \frac{\langle 0, -\sin t, -\cos t\rangle}{1}$$

(1)

3 Points

e) The curvature, κ

$$\kappa = \frac{|\vec{T}\,'|}{|\vec{r}\,'|} = \frac{1}{\sec t} = \cos t$$

(1) (2)

3 Points

f) The acceleration of the projectile written in terms of **T** and **N**. You do not have to write out **T** and **N** in your answer.

$$\vec{a} = \vec{r}\,'' = \langle 0, 0, -\sec^2 t\rangle = a_T\vec{T} + a_N\vec{N}$$

$$a_T = \vec{r}\,'' \cdot \vec{T} = \langle 0, 0, -\sec^2 t\rangle \cdot \langle 0, \cos t, -\sin t\rangle = \sec^2 t \sin t$$

(3)

6 Points

$$a_N = \vec{r}\,'' \cdot \vec{N} = \langle 0, 0, -\sec^2 t\rangle \cdot \langle 0, -\sin t, -\cos t\rangle = \sec t$$

(3)

18 Points

4. Find an equation for the plane through the point (–4, 6, 1) and containing the line

$x = 3 - t$, $y = 2 - 3t$, $z = 1 + 2t$.

10 Points

pt on line = $(3, 2, 1)$

$\vec{a} = \langle -1, -3, 2 \rangle$, $\vec{b} = \langle -7, 4, 0 \rangle$ (2)

$$\vec{a} \times \vec{b} = \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ -7 & 4 & 0 \end{vmatrix} = -8\hat{i} - 14\hat{j} - 25\hat{k}$$ (4)

plane $= \langle x+4, y-6, z-1 \rangle \cdot \langle 8, 14, 25 \rangle = 0$

$8x + 14y + 25z + 32 - 84 - 25 = 0$ (4)

-77

5. Write $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ as the sum of a vector parallel to and a vector orthogonal to $\mathbf{v} = 3\mathbf{i} + \mathbf{k}$.

10 Points

$$\text{comp}_{\vec{v}} \vec{u} = \frac{\langle 1, -1, 2 \rangle \cdot \langle 3, 0, 1 \rangle}{\sqrt{9+1}} = 5/\sqrt{10}$$

$$\text{proj}_{\vec{v}} \vec{u} = \frac{5}{\sqrt{10}} \cdot \frac{\langle 3, 0, 1 \rangle}{\sqrt{10}} = \frac{1}{2} \langle 3, 0, 1 \rangle$$

(3) (3)

$$\text{orth}_{\vec{v}} \vec{u} = \langle 1, -1, 2 \rangle - \frac{1}{2}\langle 3, 0, 1 \rangle = \langle -1/2, -1, 3/2 \rangle$$

(4)

6. At time $t = 0$ a particle has the velocity 4 **i**. At time $t = 3$ the particle is located at the point (13, –9, 25). If the particle's acceleration is given by $\mathbf{a}(t) = -2\,\mathbf{j} + 6t\,\mathbf{k}$, find parametric equations for the line tangent to the particle's curve of motion at $t = 3$.

10 Points

$\vec{r}\,'(0) = 4\hat{i}$, $\vec{r}(3) = \langle 13, -9, 25 \rangle$

$\vec{r}\,'' = -2\hat{j} + 6t\hat{k}$

$\vec{r}\,' = -2t\hat{j} + 3t^2\hat{k} + \vec{c}$ (3)

$\vec{r}\,'(0) = 4\hat{i} = \vec{c}$

so $\vec{r}\,'(t) = 4\hat{i} - 2t\hat{j} + 3t^2\hat{k}$ (2)

$\vec{r}\,'(3) = \langle 4, -6, 27 \rangle$ (2)

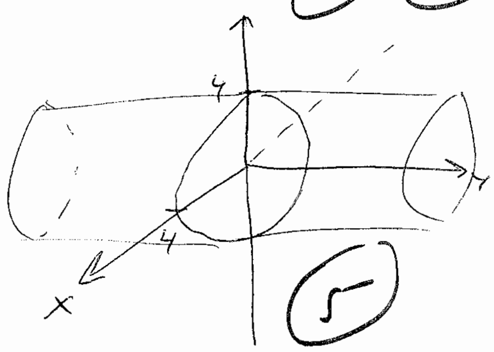
so
$x = 13 + 4t$
$y = -9 + -6t$
$z = 25 + 27t$
(3)

30 Points

7. Convert the following equation in spherical coordinates to one in rectangular coordinates and sketch its graph: $\rho^2[\sin^2\phi\cos^2\theta + \cos^2\phi] = 16$.

$x^2 + z^2 = 16$

(2) (?)



(5)

10 Points

8. Find the angle between AB and AC given the points A(1, 0, 1), B(3, 2, 0), and C(6, −3, 3).

$\vec{AB} = \langle 2, 2, -1\rangle$ (2)

$\vec{AC} = \langle 5, -3, 2\rangle$

$\cos\theta = \dfrac{\langle 2,2,-1\rangle \cdot \langle 5,-3,2\rangle}{\sqrt{4+4+1}\ \sqrt{25+9+4}} = \dfrac{2}{3\cdot\sqrt{38}}$ (3) (3)

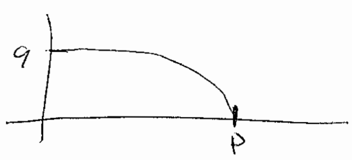
so $\theta = \cos^{-1}\dfrac{2}{3\sqrt{38}} \approx 84^\circ$, 1.46 rad

(2)

10 Points

9. A projectile is fired horizontally with initial speed of 2 ft/s from a position 9 ft above the ground. Where does the projectile hit the ground?

$\vec{V}_0 = 2\hat{i}$



$x = (V_0 \cos\alpha)t = 2t$ (1)

$y = -\frac{1}{2}(32)t^2 + 9 = 0$ (2)

$t^2 = \frac{9}{16}$

$t = 3/4$ sec (4)

so $x = 2 \cdot \frac{3}{4} = \frac{3}{2}$ ft (3)

10 Points

30 Points